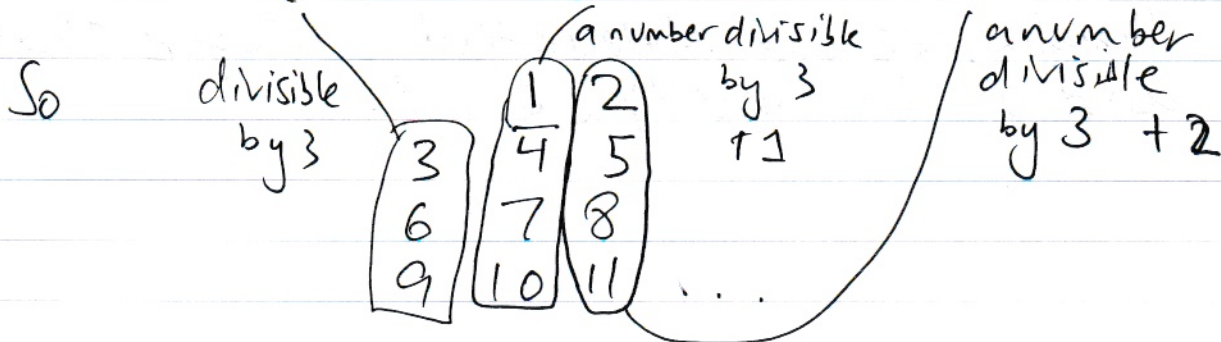


Take Three from Five

You can classify ^{integers} numbers into 3 types
 x $x+1$ $x+2$



If $x = \textcircled{1}$, $x+1 = \textcircled{2}$ and $x+2 = \textcircled{3}$
 and we needed to give 5 integers,
 the possible combinations are

- | | | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| <u>①</u> | <u>①</u> | <u>①</u> | <u>①</u> | <u>①</u> | <u>①</u> | <u>③</u> | <u>③</u> | <u>②</u> | <u>③</u> |
| <u>①</u> | <u>①</u> | <u>①</u> | <u>①</u> | <u>②</u> | <u>①</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>③</u> |
| <u>①</u> | <u>①</u> | <u>①</u> | <u>①</u> | <u>③</u> | <u>①</u> | <u>②</u> | <u>②</u> | <u>③</u> | <u>③</u> |
| <u>①</u> | <u>①</u> | <u>①</u> | <u>②</u> | <u>②</u> | <u>①</u> | <u>②</u> | <u>③</u> | <u>③</u> | <u>③</u> |
| <u>①</u> | <u>①</u> | <u>①</u> | <u>③</u> | <u>③</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>②</u> |
| <u>①</u> | <u>①</u> | <u>②</u> | <u>②</u> | <u>③</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>③</u> |
| <u>①</u> | <u>①</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>③</u> | <u>③</u> | <u>③</u> |
| <u>①</u> | <u>①</u> | <u>③</u> | <u>②</u> | <u>③</u> | <u>②</u> | <u>③</u> | <u>③</u> | <u>③</u> | <u>③</u> |
| <u>①</u> | <u>①</u> | <u>③</u> | <u>③</u> | <u>③</u> | <u>③</u> | <u>③</u> | <u>②</u> | <u>③</u> | <u>③</u> |
| <u>①</u> | <u>②</u> | <u>②</u> | <u>②</u> | <u>②</u> | | | | | |

These patterns could be arranged in a different order, but you will have the same composition [Order doesn't matter]

Now, to pick 3 of these integers and add them to get a multiple of 3, you could pick whole numbers

Why?

- or
- ① ① ①
 - ② ② ②
 - ③ ③ ③
 - ① ② ③

$$x + x + x = 3x \rightarrow 3x / 3 = x$$

$$x+1 + x+1 + x+1 = 3x+3 \rightarrow 3x+3 / 3 = x+1$$

$$x+2 + x+2 + x+2 = 3x+6 \rightarrow 3x+6 / 3 = x+2$$

$$x + x+1 + x+2 = 3x+3 \rightarrow 3x+3 / 3 = x+1$$

And we can always pick one of these set.
Look at the numbers underlined in purple

Therefore, we can always find 3 integers that
add up to a multiple of 3.