Make 37: Stage 2 and 3**

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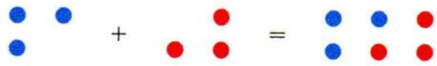
37 is prime and odd. The numbers in the bags are all odd. The question asks for 10 numbers from the bags to sum to 37. The problem is that 10 is not odd, so it is impossible to satisfy the question. This is because of the way odd and even numbers work: even x even = even and even x odd = even, but only odd x odd = odd.

Drawing dots on a page can help make this really clear, e.g.



If you sum ten odd numbers, the answer will always be even because 10 is an even number and multiplication is just repeated addition (3x1 = 1+1+1).

All even numbers are multiples of 2, so can be written as $2n$ where n is any number. An odd number is at least one more (or one less) than an even number, so can be written as $2n\pm1$, but to keep it simple, I'll stick with $2n+1$. An even number can be made up of two odd numbers (1+1 = 2) or two even numbers (2+2 = 4), but an odd number must be made up of an odd and even number (1+2 = 3).

We can prove this using algebra. It's easy to do using $2n$ and $2n+1$, but the numbers in the bags were different, so I used different variables. $2a$ and $2b$ represented even numbers. $(2a+1)$ and $(2b+1)$ represented odd numbers. $a = 1$, $b = 2$

Proof: even x even = even
e.g. 2 x 4 = 8

$2a \times 2b = 4ab$
factor out two to get: **2(2*ab*)**
2(2*ab*) is a multiple of 2, so it is an even number

Proof: even x odd = even
e.g. 2 x 5 = 10

$2a \times (2b+1) = 4ab+2a$
factor out two to get: **2(2*ab*+*a*)**
2(2*ab*+*a*) is also a multiple of 2, so it is an even number

Proof: odd x odd = odd
e.g. 3 x 5 = 15

$(2a+1) \times (2b+1) = 4ab+2a+2b+1$
factor out two to get: **2(2*ab*+*a*+*b*)+1**
2(2*ab*+*a*+*b*)+1 is NOT a multiple of 2, (because not all of the expression is a multiple of 2, only the part in the brackets), so it is an odd number

Using 1,3,5,7, the closest you can get to 37 is, for example:

7 numbers used	9 numbers used	11 numbers used	13 numbers used
(7+7+7)+(5+5+5)+1	(7+7)+(5+5+5)+(3+3)+(1+1)	(3x7)+(1x5)+(2x3)+(5x1)	(1x7)+(1x5)+(7x3)+(4x1)
(7+7+7+7+7)+(1+1)	(7+7+7)+(5+5)+3+(1+1+1)	(2x7)+(1x5)+(5x3)+(3x1)	(1x7)+(2x5)+(5x3)+(5x1)
(7+7+7)+(5+5)+(3+3)	(7+7+7)+5+(3+3+3)+(1+1)	(1x7)+(1x5)+(8x3)+(1x1)	(2x7)+(1x5)+(4x3)+(6x1)
	(7+7)+(5+5)+(3+3+3+3)+1	(1x7)+(2x5)+(6x3)+(2x1)	

This shows that an odd number of odd numbers will sum to an odd number. If ten numbers are to sum to 37, at least one of those numbers must be even, e.g. {1,2,3,5} could give you: (5x5) + (3x3) + (1x2) + (1x1) = 37, or say {1,3,4,7} could give you (3x7) + (1x4) + (3x3) + (3x1) = 37.

You can play around with the numbers and come up with quite a few different options, but in order to sum ten numbers to 37, at least one of the numbers needs to be even.

This leaves you with different combinations of (odd x odd = odd) + (odd x even = even), which you can sum to make 37. All of the sums will be in the form (odd + even = odd), because 37 is odd. For example:

If you take the set {1,2,3,5}, then	(5x5)	+	(3x3)	+	(1x2)	+	(1x1)	=	37
becomes	(OxO)	+	(OxO)	+	(OxE)	+	(OxO)		
which simplifies to	O	+	O	+	E	+	O	=	O

Again algebra can prove this: e.g. $a = 1$, $b = 2$

Proof: odd + even = odd

	odd x odd = **odd**	odd x even = **even**	**odd + even = odd**
Example	3 x 5 = 15	2 x 5 = 10	15+10 = 25
Algebra:	$(2a+1) \times (2b+1) = 4ab+2a+2b+1$	$2a \times (2b+1) = 4ab+2a$	$4ab+2a+2b+1$ $+ \underline{4ab+2a}$ $8ab+4a+2b+1$, or $\mathbf{2(4ab+2a+b)+1}$