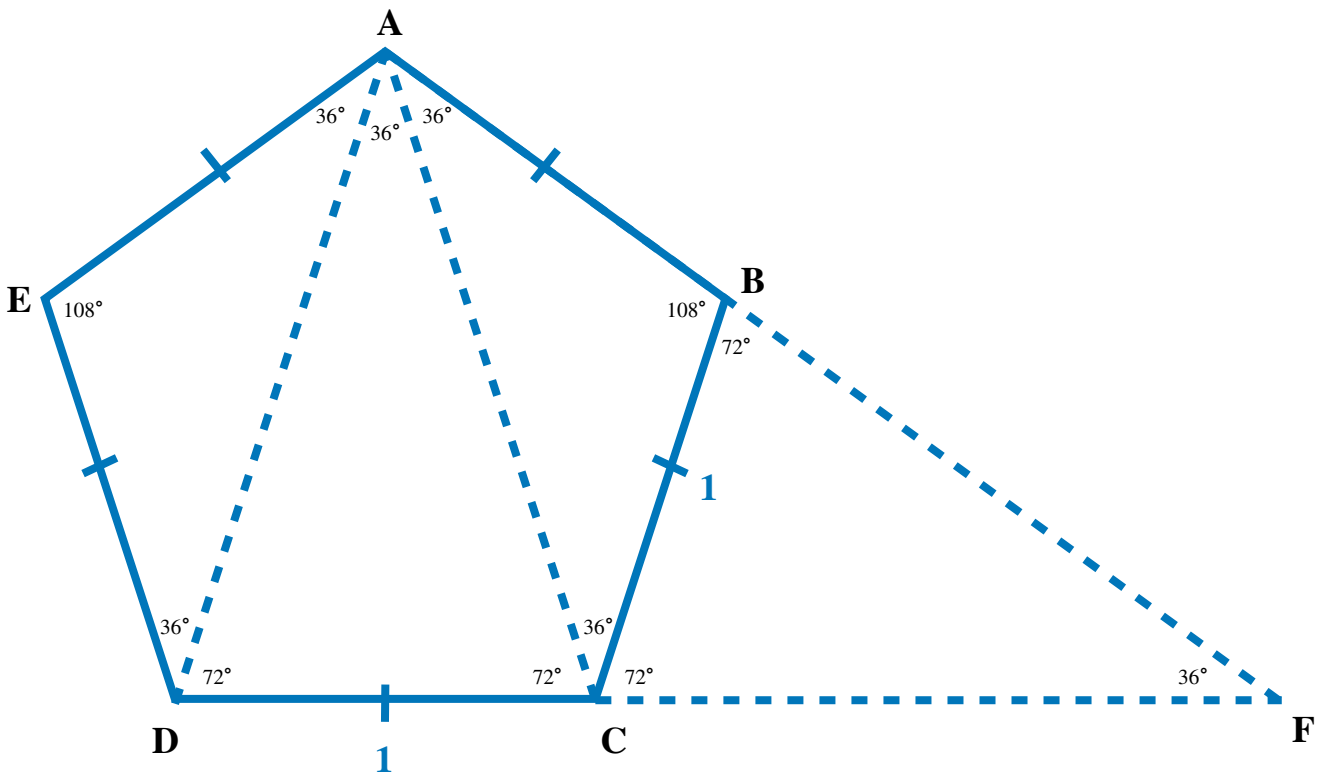
Pentakite - Mahdi Raza

Proving $\triangle BFC \cong \triangle DAC$



After a bit of angle chasing, we get the following angles.

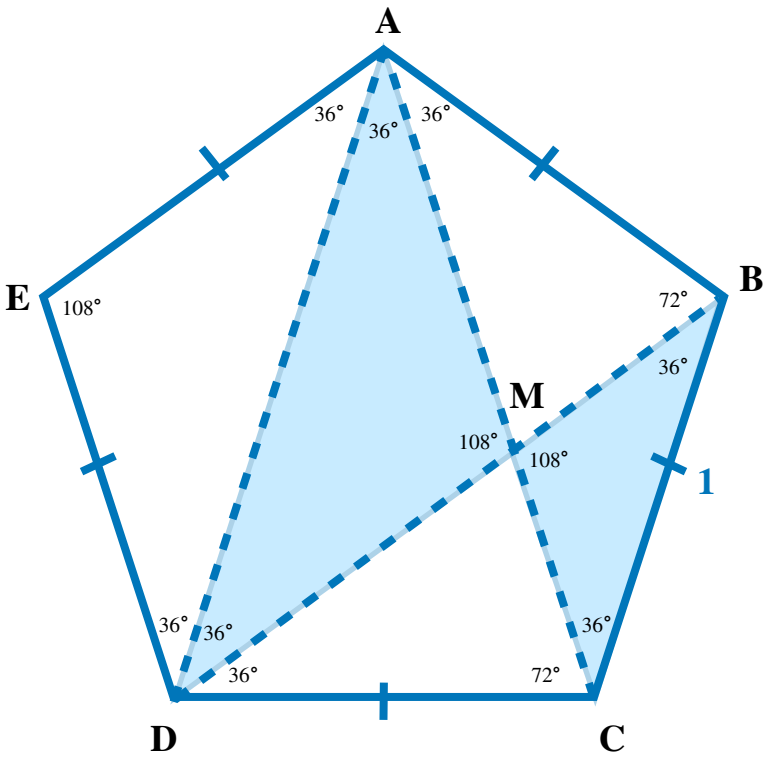
$$\angle CDA = \angle CBF = 72^\circ$$

$$\angle ACD = \angle FBC = 72^\circ$$

$$BC = DC = 1$$

Using the ASA congruency criterion, we have $\triangle BFC \cong \triangle DAC$

Proving $\triangle BFC \cong \triangle DAC$



Let $AD = x$

Now, all similar diagonals have same length. So, $AC = x$ and $BD = x$

Quadrilateral AEDM is a parallelogram. Since $DE = 1$, $AM = 1$ too

So, $BM = x - 1$

And $BC = 1$ since this is a regular pentagon with side length 1

Using AA criterion for similarity, $\triangle BMC \sim \triangle AMD$

$$\frac{BM}{BC} = \frac{AM}{AD}$$

$$\frac{x-1}{1} = \frac{1}{x}$$

$$x^2 - x - 1 = 0$$

$$x = \phi \qquad \because x > 0$$

So, $DA = \phi$, the golden ratio!

Pentagon Fractal