1.

a)

$$n(n+2)+1 = n^2+2n+1 = (n+1)^2$$

b)

$$n(n+4)+4 = n^2+4n+4 = (n+2)^2$$

c)

$$n(n+6)+9 = n^2+6n+9 = (n+3)^2$$

d)

We claim the answer is k^2.

$$n(n+2k)+k^2 = n^2+2kn+k^2$$

$= (n+k)^2$, which is indeed a square.

As no assumptions have been made on n or k, this is the general answer.

2.

a)

We claim the answer is the square of (the product of (the middle two numbers minus 1)).

Consider:

$(n-1)n(n+1)(n+2)+1$

$= (n^2-1)(n^2+2n)+1$

$= n^4+2n^3-n^2-2n+1$

$= (n(n+1)-1)^2$

b) We have

$(n-3)(n-1)(n+1)(n+3)+16$

$= (n^2-9)(n^2-1)+16$

$= n^4-10n^2+25$

$= (n^2-5)^2$. So the result will

always be the mean of the four numbers, squared, minus 5, then squared again.

c) We have

$$(n-3)\,n\,(n+3)(n+6)+81$$

$$=(n^2-9)(n^2+6n)+81$$

$$=n^4+6n^3-9n^2-54n+81$$

$$=\left(n(n+3)-9\right)^2$$

(testing)

1	4	7	10	19^2
2	5	8	11	31^2
3	6	9	12	45^2
4	7	10	13	61^2

d)

We claim that adding k^4 does the trick.

Consider k is even

$$\left(n-\frac{3k}{2}\right)\left(n-\frac{k}{2}\right)\left(n+\frac{k}{2}\right)\left(n+\frac{3k}{2}\right)+k^4$$

$$=\left(n^2-\frac{9k^2}{4}\right)\left(n^2-\frac{k^2}{4}\right)$$

$$=n^4-10\frac{k^2}{4}n^2+\frac{9k^4}{16}+k^4$$

$$=n^4-10\frac{k^2}{4}n^2+\frac{25k^4}{16}$$

$$=\left(n^2-\frac{5k^2}{4}\right)^2$$

Otherwise if k is odd

$$(n-k)n(n+k)(n+2k)+k^4$$

$$=(n^2-k^2)(n^2+2kn)+k^4$$

$$=n^4+2kn^3-k^2n^2-2k^3n+k^4$$

$$= \left(n(n+k) - k^2 \right)^2$$