

$$\sqrt{x} + \frac{1}{\sqrt{x}} < 4$$

Since the expression contains \sqrt{x} and $\frac{1}{\sqrt{x}}$, $x > 0$ for inequalities to hold any meaning; if $x < 0$, there would be complex numbers involved, which do not conform to inequalities; if $x = 0$, $\frac{1}{\sqrt{x}}$ is undefined.

$$\sqrt{x} + \frac{1}{\sqrt{x}} < 4$$

$$() \times \sqrt{x}$$

The inequality sign is unchanged because $\sqrt{x} > 0$

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \times \sqrt{x} < 4\sqrt{x}$$

$$x + 1 < 4\sqrt{x}$$

$$()^2$$

$$(x + 1)^2 < (4\sqrt{x})^2$$

Since we know that if $|a| < |b|$, it implies $a^2 < b^2$. Our assumption of $x+1 > 0$ and $4\sqrt{x} > 0$ is justified because $x > 0$

$$(x + 1)^2 < 16x$$

$$x^2 + 2x + 1 < 16x$$

$$() - 16x$$

$$x^2 + 2x + 1 - 16x < 0$$

$$x^2 - 14x + 1 < 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(1)}}{2(1)} = \frac{14 \pm \sqrt{192}}{2} = \frac{14 \pm \sqrt{64 \times 3}}{2}$$

$$x = \frac{14 \pm \sqrt{64}\sqrt{3}}{2} = \frac{14 \pm 8\sqrt{3}}{2} = 7 \pm 4\sqrt{3}$$

By drawing the number line, we obtain:

$$7 - 4\sqrt{3} < x < 7 + 4\sqrt{3}$$