POLITE NUMBERS

~~20~~ $10+11 = 21$

$1+2+3+4 = 10$

$(1+2+3+4)+(5+6) = 21$

$(n)+(n+1) = 2n+1 \Rightarrow$ all odd numbers

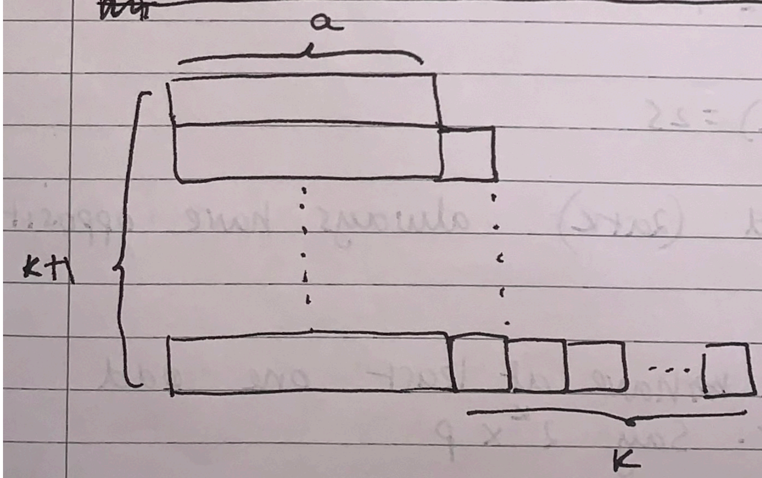
63 is odd $\Rightarrow 63 = 2n+1 \therefore \underline{31+32} \therefore$ Polite

$(n)+(n+1)+(n+2) = 3n+3 = 3(n+1)$

$\therefore$ all multiples of 3 are also polite no.

$(n)+(n+1)+(n+2)+(n+3)+(n+4) = 5n+10$
$= 5(n+2)$

$\therefore$ all multiples of ~~3~~ 5 are also polite



$= \boxed{(k+1)(a) + \frac{(k)(k+1)}{2}}$

geometric

$(a) + (a+1) + \cdots (a+k)$

$= (k+1)a + (1+2+3 \cdots k)$

$= \boxed{(k+1)a + \frac{(k)(k+1)}{2}}$ algebraic

∴ The sum of $(k+1)$ consecutive numbers is this, where the first number is a and last numbers is a+k

The sum can be ~~as~~ manipulated as such:

$(k+1)\left(a + \frac{k}{2}\right) = S$

$(k+1)\left(\frac{2a+k}{2}\right) = S$

$(k+1)(2a+k) = 2S$

$(k+1)$ and $(2a+k)$ always have opposite parity.

∴ S will ~~be~~ have at least one odd factor. Say $2^k \times p$

$S = 2^{K} p$

$(K+1)\left(\frac{2a+K}{2}\right) = 2^{K} \times p$ where p is add ~~integer~~ number

<u>Fact:</u> Any number can be represented in the form of $2^{k} \times m$ where m is odd number or $m = 1$

Here p is strictly odd and not equal to 1

$\therefore$ any number of the form 2^{K} can't be polite

Except that, all numbers are polite.