We claim the rule is

$$n \text{ is polite} \iff n \neq 2^k \text{ for any } k \in \mathbb{Z} \geq 0.$$

If n is polite, we have

$$n = r + (r+1) + \dots + (r+m) \quad \text{for some } r, m \in \mathbb{N}$$

$$= (m+1)r + \frac{m(m+1)}{2}$$

$$= (m+1)\left(r + \tfrac{1}{2}m\right) \quad \circledast \ \underbrace{\text{(generic form of a polite number)}}$$

if m is even, then n has an odd factor — $m+1$ — so isn't a power of 2. if m is odd, we can write

$$(m+1)\left(r + \tfrac{1}{2}m\right) = \left(\frac{m+1}{2}\right)(2r+m),$$

so n still has an odd factor — $2r+m$ — and again cannot be a power of two.

We have shown

$$n \text{ is polite} \Rightarrow n \neq 2^k \text{ for any } k \in \mathbb{Z} \geq 0.$$

To show the other direction.

Consider the general form of a polite number found earlier

$$\frac{1}{2}(m+1)(2r+m)$$

if n is odd, we can set $m=1$ to get a polite number of the form

$$2r+1 \quad \text{(generic odd number)}$$

for which we can choose a value of r to set n.

if n is even (but not a power of 2), we can write

$$n = 2^u v \qquad u \in \mathbb{N}$$

where v is odd and greater than 1.

Setting $m = v-1$

we get a polite number of the form

$$\frac{1}{2}v\left(2r+v-1\right) \quad \text{(using *)}$$

$$=\frac{1}{2}v\left(2r+v-1\right)$$

we need r such that

$$2r+v-1=2^{m+1}$$

$$\Leftrightarrow r=\frac{2^{m+1}-(v-1)}{2}$$

$v-1$ is even, so clearly such an integral value of r exists.

Therefore, for any $n \neq 2^k$, there exists a sequence of $m+1$ consecutive integers, the smallest of which is r, which sum to give n. So n is polite.

We have therefore shown

$$n \text{ is polite} \Leftrightarrow n \neq 2^k \text{ for any } k \in \mathbb{Z} \geq 0.$$