Farey Sequences

$F_1 = \frac{0}{1} \quad \frac{1}{1}$

$F_2 = \frac{0}{1} \quad \frac{1}{2} \quad \frac{1}{1}$

$F_3 = \frac{0}{1} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{1}$

$F_4 = \frac{0}{1} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{1}{1}$

$F_5 = \frac{0}{1} \quad \frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{2}{5} \quad \frac{1}{2} \quad \frac{3}{5} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{1}{1}$

If ~~its~~ n is a prime ~~denominator~~ in the sequence of F_n, $(n-1)$ fractions are added:

$F_{11} - F_{10} = \frac{1}{11} \quad \frac{2}{11} \quad \frac{3}{11} \quad \frac{4}{11} \quad \frac{5}{11} \quad \frac{6}{11} \quad \frac{7}{11} \quad \frac{8}{11} \quad \frac{9}{11} \quad \frac{10}{11}$. (10 fractions)

If its not a prime then any factors of n can be simplified.

A = the set of fractions added with each denominator, N

$A = \{ x \in \mathbb{N}, x \leq N \}$

$B \subseteq A \quad B = \{\text{factors of } N\}$

$\therefore$ number of new fractions $= n(B')$

$n(B') =$ ~~even~~ $2i$ (i is an integer) $\because$ the fractions can always be paired up to add up to 1

e.g. $\frac{1}{5}, \frac{4}{5} \quad \frac{2}{5}, \frac{3}{5}$

the only time you can't pair up is when the numerator is half of the denominator which is $\frac{1}{2}$ established in F_2.