(a)

A set of integers $A=\{-2,-1,0,1,2\}$ with the operation of addition satisfies the four given conditions:

+	-2	-1	0	1	2
-2	-4	-3	-2	-1	0
-1	-3	-2	-1	0	1
0	-2	-1	0	1	2
1	-1	0	1	2	3
2	0	1	2	3	4

Identity element: 0
Inverse element: Yes, every value has an inverse element to get the identity element.
Commutative: Yes, when draw a diagonal from top left to bottom right, the table is symmetrical
Associative: Yes, since this is an addition operation, the final value would be the same regardless of the way it is solved.

Therefore, set A **is a group.**

A set of natural integers $B=\{1,2,3,4,5\}$ with the operation of subtraction does not satisfy the four given conditions:

-	1	2	3	4	5
1	0	-1	-2	-3	-4
2	1	0	-1	-2	-3
3	2	1	0	-1	-2
4	3	2	1	0	-1
5	4	3	2	1	0

Identity element: 0
Inverse element: No, not every value has an inverse element to get the identity element.
Commutative: No, when draw a diagonal from top left to bottom right, the table is not symmetrical
Associative: No, since this is an subtraction operation, the order of subtraction would affect the final value.

Therefore, set B **is not a group.**

(b)
A set of positive rational numbers $C = \{0, 0.5, 1, 1.5, 2\}$ with the operation of multiplication satisfies the four given conditions:

x	**0**	**0.5**	**1**	**1.5**	**2**
0	0	0	0	0	0
0.5	0	0.25	0.5	0.75	1
1	0	0.5	1	1.5	2
1.5	0	0.75	1.5	2.25	3
2	0	1	2	3	4

Identity element: 1
Inverse element: Yes, every value has an inverse element to get the identity element.
Commutative: Yes, when draw a diagonal from top left to bottom right, the table is symmetrical
Associative: Yes, since this is an multiplication operation, the final value would be the same regardless of the way it is solved.

Therefore, set C **is a group when multiplying.**

A set of positive rational numbers $C = \{0, 0.5, 1, 1.5, 2\}$ with the operation of division does not satisfy the four given conditions:

/	**0**	**0.5**	**1**	**1.5**	**2**
0	0	0	0	0	0
0.5	0	1	0.5	0.3333	0.25
1	0	2	1	0.6666	0.5
1.5	0	3	1.5	1	0.75
2	0	4	2	1.3333	1

Identity element: 1
Inverse element: No, not every value has an inverse element to get the identity element.
Commutative: No, when draw a diagonal from top left to bottom right, the table is not symmetrical
Associative: No, since this is an division operation, the final value would be affected by the order of the operations.

Therefore, set C **is not a group when dividing.**

(c)
A set of positive integers $D = \{1,2,3,4,5\}$ with the operation of multiplication does not satisfy the four given conditions:

x	**1**	**2**	**3**	**4**	**5**
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Identity element: 1
Inverse element: No, not every value has an inverse element to get the identity element.
Commutative: Yes, when draw a diagonal from top left to bottom right, the table is not symmetrical
Associative: Yes, since this is an multiplication operation, the final value would not be affected by the order of the operations.

Therefore, set D **is not a group when dividing because it does not satisfy one condition.**

(d)
A set of positive even integers $E = \{2,4,6,8,10\}$ with the operation of multiplication does not satisfy the four given conditions:

x	**2**	**4**	**6**	**8**	**10**
2	4	8	12	16	20
4	8	16	24	32	40
6	12	24	36	48	60
8	16	32	48	64	80
10	20	40	60	80	100

Identity element: None
Inverse element: No identity element, and so no inverse element
Commutative: Yes, when draw a diagonal from top left to bottom right, the table is not symmetrical
Associative: Yes, since this is an multiplication operation, the final value would not be affected by the order of the operations.

Therefore, set E **is not a group when dividing because it does not satisfy two conditions.**

(e)

$m * n = m + n + 1$ is a group

$m * e = m$ (when e is the identity)

$m * e \Rightarrow m + e + 1 = m$

$\therefore e = -1$ (identity element)

$m * m' = m + m' + 1$

$m * m' = e \Rightarrow m + m' + 1 = -1$

$\therefore m' = -2 - m$ (inverse element)

(f)

$m * n = m + (-1)^m n$ is a group

$m * e = m$ (when e is the identity)

$m * e \Rightarrow m + (-1)^m e = m$

$\therefore e = 0$ (identity element)

$m * m' = m + (-1)^m m'$

$m * m' = e \Rightarrow m + (-1)^m m' = 0$

$(-1)^m m' = -m$

$\therefore m' = -m$ (inverse element, when m is a positive integer)

$m' = m$ (inverse element, when m is a negative integer)

(g)

$x * y = xy + x + y$ is a group

$x * e = x$ (when e is the identity)

$x * e \Rightarrow xe + x + e = x$

$xe + e = 0$

$e(x + 1) = 0$

$\therefore e = 0$ (identity element)

$x * x' = xx' + x + x'$

$x * x' = e \Rightarrow xx' + x + x' = 0$

$xx' + x' = -x$

$x'(x + 1) = -x$

$\therefore x' = \dfrac{-x}{(x+1)}$ (inverse element)