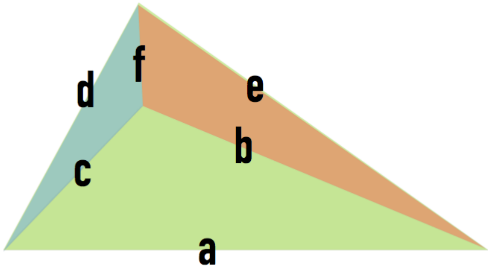
Live problem : proof : Building Tetrahedra - nrich　　　　　　　　　Name : Makalp Patel

To approach this problem, a simple diagrammatic representation of a tetrahedron will be used. Each edge of the tetrahedron is labelled with an alphabet from a to f. Refer to the diagram for the below calculations.



An equation for the perimeter of each triangular side is formed using the labelled variables, as shown in the figure. To fulfil the condition that all triangle have the same perimeter, perimeter of each triangle is equated to a constant, k.

Therefore,

$a + b + c = k$ - - - (1)
$a + d + e = k$ - - - (2)
$b + e + f = k$ - - - (3)
$c + d + f = k$ - - - (4)

solving two equations simultaneously to eliminate a variable gives the following 6 equations

$b + c = d + e$ - - - (1-2)
$a + d = b + f$ - - - (2-3)
$b + e = c + d$ - - - (3-4)
$a + c = e + f$ - - - (1-3) - - - (A)
$a + b = d + f$ - - - (1-4) - - - (B)
$a + e = c + f$ - - - (2-4)

Subtracting (B) from (A) to eliminate f gives

$a + c = e + f$ - - - (A)
\- $a + b = d + f$ - - - (B)

$c - b = e - d$ - - - (A-B)

When this is added to equation (1-2)

$b + c = d + e$ - - - (1&2)
\+ $- b + c = - d + e$ - - - (A-B)

$2c = 2e$

Therefore

$c = e$

Similarly by using the above deduced result and the equations, relationships for a, b, d and f were also determined.

$a = f$, $b = d$ and $c = e$

Therefore, there are infinite possible tetrahedrons whose all 4 triangular sides have the same perimeter. All dimensions that follow the above mentioned condition is a possible solution i.e. $a = f$, $b = d$ and $c = e$. Thus, the triangular sides can be equilateral, isosceles and scalene.