

Let  $r = 1$  ruby,  $s = 1$  sapphire,  $p = 1$  pearl,  $d = 1$  diamond.

The four jewellers started with:  $8r$ ,  $10s$ ,  $100p$ ,  $5d$

After exchanging gifts they had:  $5r + s + p + d$ ,  $7s + r + p + d$ ,  $97p + r + s + d$ ,  $2d + r + s + p$

Using the associative law, this can be rewritten as:

$$4r + (r + s + p + d), \quad 6s + (r + s + p + d), \quad 96p + (r + s + p + d), \quad d + (r + s + p + d)$$

Simplify to get  $4r$ ,  $6s$ ,  $96p$ ,  $d$  and these are all equal (given).

Construct a table:

$$\begin{array}{llll} r = \frac{3s}{2} & s = \frac{2r}{3} & p = \frac{r}{24} & d = 4r \\ r = 24p & s = 16p & p = \frac{s}{16} & d = 6s \\ r = \frac{d}{4} & s = \frac{d}{6} & p = \frac{d}{96} & d = 96p \end{array}$$

We have now calculated the relative value of each gem. To avoid fractions, and make it neater, express everything in terms of  $p$ . Using this, we can work out how much each jeweller gained or lost.

	Start value in pearls	End value in pearls	Gain in pearls
Ruby collection	$8r = 192p$	$4r + (r + s + p + d) = 233p$	$233 - 192 = +41p$
Sapphire collection	$10s = 160p$	$6s + (r + s + p + d) = 233p$	$233 - 160 = +73p$
Pearl collection	$100p$	$96p + (r + s + p + d) = 233p$	$233 - 100 = +133p$
Diamond collection	$5d = 480p$	$d + (r + s + p + d) = 233p$	$233 - 480 = -247p$

At the start, the collections were worth:  $480p > 192p > 160p > 100p \quad \therefore \quad 5d > 8r > 10s > 100p$

But at the end the values were all equal.

Therefore the pearl collector, which started out with the lowest relative value, gained the most; and the diamond collector, who started out the highest, lost the most.

The gains are:  $133 > 73 > 41 > -247$ , therefore:

pearl collector's gain  $>$  sapphire collector's gain  $>$  ruby collector's gain  $>$  diamond collector's gain

$(r + s + p + d) = 137p$  and 137 is prime. After the gem swaps are complete, each collection is worth  $233p$ , and 233 is prime. 41 is prime as is 73.

I wanted to calculate a percentage increase or decrease in value, but you don't end up with a neat solution. I thought about substituting a value for  $k$  into the equations, but the relationships remain the same. If you leave the percentage in (equivalent) fraction form, it doesn't change the result.

So for fun, I broke it down using simple addition and subtraction, but still using pearls as a common base:

	$8r$	$10s$	$100p$	$5d$
Equivalent pearl value at the start	$192p$	$160p$	$100p$	$480p$
Ruby exchange	$\frac{-72p}{120p}$	$\frac{+24p}{184p}$	$\frac{+24p}{124p}$	$\frac{+24p}{504p}$
Value after Ruby swap	$120p$	$184p$	$124p$	$504p$
Sapphire exchange	$\frac{+16p}{136p}$	$\frac{-48p}{136p}$	$\frac{+16p}{140p}$	$\frac{+16p}{520p}$
Value after ruby and sapphire swap	$136p$	$136p$	$140p$	$520p$
Pearl exchange	$\frac{+p}{137p}$	$\frac{+p}{137p}$	$\frac{-3p}{137p}$	$\frac{+p}{521p}$
Value after ruby, sapphire and pearl swap	$137p$	$137p$	$137p$	$521p$
Diamond exchange	$\frac{+96p}{233p}$	$\frac{+96p}{233p}$	$\frac{+96p}{233p}$	$\frac{-288p}{233p}$
Equivalent pearl value at the end	$233p$	$233p$	$233p$	$233p$

It's easy to see how the values equalise.