

Let us first understand the combination that form addition of odd and even

$$\text{EVEN} + \text{EVEN} = \text{EVEN}$$

$$\text{ODD} + \text{EVEN} = \text{ODD}$$

$$\text{ODD} + \text{ODD} = \text{EVEN}$$

- For set A), there are 5 balls. The total number of pairs can be determined using

$$\binom{5}{2} = 10$$

The 10 pairs formed are listed below along with their result on addition

$$2 + 3 = 5 \quad (\text{Odd})$$

$$3 + 5 = 8 \quad (\text{Even})$$

$$2 + 4 = 6 \quad (\text{Even})$$

$$3 + 6 = 9 \quad (\text{Odd})$$

$$2 + 5 = 7 \quad (\text{Odd})$$

$$4 + 5 = 9 \quad (\text{Odd})$$

$$2 + 6 = 8 \quad (\text{Even})$$

$$4 + 6 = 10 \quad (\text{Even})$$

$$3 + 4 = 7 \quad (\text{Odd})$$

$$5 + 6 = 11 \quad (\text{Odd})$$

Probability of winning =  $4 / 10$

You would win about 40% of the time

- For set B), there are 5 balls. The total number of pairs can be determined using

$$\binom{5}{2} = 10 \quad \text{The 10 pairs formed are listed below along with their result on addition}$$

$$1 + 3 = 4 \quad (\text{Even})$$

$$3 + 6 = 9 \quad (\text{Odd})$$

$$1 + 5 = 6 \quad (\text{Even})$$

$$3 + 7 = 10 \quad (\text{Even})$$

$$1 + 6 = 7 \quad (\text{Odd})$$

$$5 + 6 = 11 \quad (\text{Odd})$$

$$1 + 7 = 8 \quad (\text{Even})$$

$$5 + 7 = 12 \quad (\text{Even})$$

$$3 + 5 = 8 \quad (\text{Even})$$

$$6 + 7 = 13 \quad (\text{Odd})$$

Probability of winning =  $6 / 10$

You would win about 60% of the time

- For set C), there are 6 balls. The total number of pairs can be determined using

$$\binom{6}{2} = 15$$

The 15 pairs formed are listed below along with their result on addition

$$2 + 3 = 5 \text{ (Odd)} \quad 3 + 4 = 7 \text{ (Odd)} \quad 4 + 6 = 10 \text{ (Even)}$$

$$2 + 4 = 6 \text{ (Even)} \quad 3 + 5 = 8 \text{ (Even)} \quad 4 + 8 = 12 \text{ (Even)}$$

$$2 + 5 = 7 \text{ (Odd)} \quad 3 + 6 = 9 \text{ (Odd)} \quad 5 + 6 = 11 \text{ (Odd)}$$

$$2 + 6 = 8 \text{ (Even)} \quad 3 + 8 = 11 \text{ (Odd)} \quad 5 + 8 = 13 \text{ (Odd)}$$

$$2 + 8 = 10 \text{ (Even)} \quad 4 + 5 = 9 \text{ (Odd)} \quad 6 + 8 = 14 \text{ (Even)}$$

Probability of winning =  $7 / 15$

You would win about 46.67% of the time

- For any general set containing 'o' odd numbers and 'e' even numbers

$$\text{Total number of pairs formed} = \binom{o + e}{2}$$

Total favourable even pairs can be formed by choosing pairs of 'o' odd numbers and  
can be formed by choosing pairs of 'e' even numbers

$$\text{Favourable outcomes} = \binom{o}{2} + \binom{e}{2}$$

Probability = Favourable / Total

$$= \frac{\binom{o}{2} + \binom{e}{2}}{\binom{o + e}{2}}$$

Let us verify the general strategy for Set A), Set B) and Set C)

$$\text{For Set A) :- 2 odd, 3 even} \quad \frac{\binom{2}{2} + \binom{3}{2}}{\binom{5}{2}} = \frac{1 + 3}{10} = \frac{4}{10}$$

$$\text{For Set B) :- 4 odd, 1 even} \quad \frac{\binom{4}{2} + \binom{1}{2}}{\binom{5}{2}} = \frac{6 + 0}{10} = \frac{6}{10}$$

$$\text{For Set C) :- 2 odd, 4 even} \quad \frac{\binom{2}{2} + \binom{4}{2}}{\binom{6}{2}} = \frac{1 + 6}{15} = \frac{7}{15}$$

Similarly, we can continue for set D) as well

$$\text{For Set D) :- 2 odd, 4 even} \quad \frac{\binom{1}{2} + \binom{5}{2}}{\binom{6}{2}} = \frac{0 + 10}{15} = \frac{10}{15}$$

Thus, we should select set D) because it has the highest probability of winning

How to maximise the probability of winning, When favourable outcomes = total outcomes that is to say that all outcomes are favourable and we win if any is even

When either 'o' or 'e' is 0 we maximise our chances of winning. Thus when there are all odd or all even balls in the set you always have a guarantee to win.

This can be shown mathematically as well.

$$\frac{\binom{0}{2} + \binom{e}{2}}{\binom{0+e}{2}} = \frac{\binom{e}{2}}{\binom{e}{2}} = 1$$

$$\frac{\binom{o}{2} + \binom{0}{2}}{\binom{o+0}{2}} = \frac{\binom{o}{2}}{\binom{o}{2}} = 1$$