Different by one - Tough nut

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Let the two rods be of difference d. Thus let the lengths be $k, k+d$.

Both rods cannot be even

Difference of 1:

Let there be (x) number of k rods and (y) such $k+d$ length rods.

Then the difference of 1 can be either of the two

$$(x)k - y(k+d) = 1 \qquad \text{OR} \qquad y(k+d) - (x)k = 1$$

We can make one of the coefficients as the subject of equation. Let that be x, because putting values of y and simplifying the term will reduce the possibilities.

$$\begin{aligned} xk - y(k+d) &= 1 & \text{OR} && y(k+d) - xk &= 1 \\ xk &= 1 + y(k+d) & \text{OR} && -xk &= 1 - y(k+d) \\ x &= \frac{1 + y(k+d)}{k} & \text{OR} && x &= \frac{y(k+d) - 1}{k} \\ x &= \frac{1 + y\color{red}{(k+d)}}{\color{red}{k}} & \text{OR} && x &= \frac{-1 + y\color{red}{(k+d)}}{\color{red}{k}} \end{aligned}$$

The values of Red (k, k+d) are known because they are the length of the two rods

Algorithm:

Step 1: Obtain the end result in terms of this form after simplifying

$$x = \frac{1 + y(k + d)}{k} \quad \text{OR} \quad x = \frac{-1 + y(k + d)}{k}$$

$$x = \frac{yk}{k} + \frac{yd + 1}{k} \quad \text{OR} \quad x = \frac{yk}{k} + \frac{yd - 1}{k}$$

$$x = y + \frac{yd + 1}{k} \quad \text{OR} \quad x = y + \frac{yd - 1}{k}$$

Step 2: Use trial and error for y and check whether $\frac{yd + 1}{k}$ and $\frac{yd - 1}{k}$ is an integer where d and k are known

Step 3: Substitute to find value of x

$$x = y + \frac{yd + 1}{k} \quad \text{OR} \quad x = y + \frac{yd - 1}{k}$$

Thus our equation is solved for two pairs (x_1, y_1) and (x_2, y_2)

<u>To get infinite pairs</u>

Step 4: Find the LCM of k and $k + d$. If they are mutually prime, the LCM is $k(k + d)$

Step 5:

$$\left(x_1 + \frac{\text{LCM}}{k}, y_1 + \frac{\text{LCM}}{k + d}\right) \text{ and } \left(x_2 + \frac{\text{LCM}}{k}, y_2 + \frac{\text{LCM}}{k + d}\right)$$

If they are prime:

$$\left(x_1 + k + d, y_1 + k\right) \text{ and } \left(x_2 + k + d, y_2 + k\right)$$

Step 6: Adjust 'n' to any value and get infinite pairs

$$\left(x_1 + n \cdot \frac{\text{LCM}}{k}, y_1 + n \cdot \frac{\text{LCM}}{k + d}\right) \text{ and } \left(x_2 + n \cdot \frac{\text{LCM}}{k}, y_2 + n \cdot \frac{\text{LCM}}{k + d}\right)$$

If prime: $\left(\left(x_1 + n \cdot (k + d)\right), \left(y_1 + n \cdot (k)\right)\right)$ and $\left(\left(x_2 + n \cdot (k + d)\right), \left(y_1 + n \cdot (k)\right)\right)$

Algorithm - Simple Example:

Let the two rods be of difference 2. Thus let the lengths be 3,5

Step 1: Obtain the end result in terms of this form after simplifying

$$x = y + \frac{2y+1}{3} \qquad \text{OR} \qquad x = y + \frac{2y-1}{3}$$

Step 2: Use trial and error for y and check whether $\frac{2y+1}{3}$ and $\frac{2y-1}{3}$ is an integer where d and k are known

Simple enough to see that: $\frac{2(1)+1}{3} = 1$ and $\frac{2(2)-1}{3} = 1$

$y_1 = 1$ and $y_2 = 2$

Step 3: Substitute to find value of x

$$x = 1 + \frac{1(2)+1}{3} \qquad \text{OR} \qquad x = 2 + \frac{2(2)-1}{3}$$

$$x_1 = 2 \qquad \text{OR} \qquad x_2 = 3$$

Thus our equation is solved for two pairs (2,1) and (3,2)

<u>To get infinite pairs</u>

Step 4: Find the LCM of k and $k + d$

LCM of 3 and 5 = 15 and they are prime

Step 5:

$$(2+5, 1+3) \text{ and } (3+5, 2+3)$$
$$=(7,4) \text{ and } (8,5)$$

Algorithm - Complex Example:

Let the two rods be of difference 23. Thus let the lengths be 70,93.

Step 1: Obtain the end result in terms of this form after simplifying

$$x = y + \frac{23y + 1}{70} \qquad \text{OR} \qquad x = y + \frac{23y - 1}{70}$$

Step 2: Use trial and error for y and check whether $\frac{23y + 1}{70}$ and $\frac{23y - 1}{70}$ is an integer where d and k are known

$$\frac{23(3) + 1}{70} = 1 \text{ and } \frac{23(67) - 1}{70} = 22$$

$y_1 = 3$ and $y_2 = 67$

Step 3: Substitute to find value of x

$$x_1 = 4 \qquad \text{OR} \qquad x_2 = 89$$

Thus our equation is solved for two pairs (4,3) and (89,67)

<u>To get infinite pairs</u>

Step 4: Find the LCM of k and $k + d$

LCM of 70 and 93 = 6510 and they are prime

Step 5:

$$(4 + 93, 3 + 70) \text{ and } (89 + 93, 67 + 70)$$
$$=(97,73) \text{ and } (182,137)$$

CHECK:

$$97 \times 70 - 73 \times 93 \equiv 1$$

$$137 \times 93 - 182 \times 70 \equiv 1$$