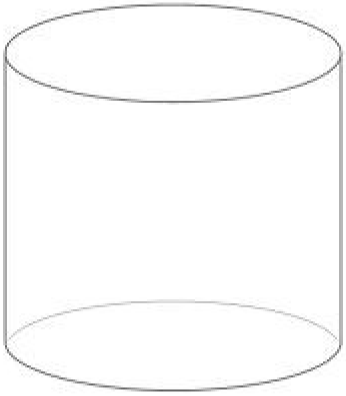
Tin Tight

What are the most efficient proportions for a tin of paint ?
"Efficient" could mean different things from different points of view - maybe start by trying to minimise the amount of material needed to make a tin with that volume.

What is the best diameter and height for a tin that contains 1 litre of paint?

What about a 5 litre tin?



Solution

Volume of Cylinder = $\pi r^2 h = 1$

Surface area of Cylinder = $2\pi r^2 + 2\pi r h$

For Volume: $\pi r^2 h = 1$: $h = \frac{1}{\pi r^2}$

Surface area = $2\pi r^2 + 2\pi r h$

$$2\pi r^2 + 2\pi r\left(\frac{1}{\pi r^2}\right)$$

$$2\pi r^2 + \frac{2\pi r}{\pi r^2}$$

$$2\pi r^2 + \frac{2}{r}$$

$$2\pi r^2 + 2r^{-1}$$

$$\frac{\delta y}{\delta x} = 4\pi r - 2r^{-2}$$

At the minimum point, gradient = 0

We know this is a minimum as $\frac{\delta^2 y}{\delta x^2} = 4\pi + \frac{4}{r^{-3}}$ must be positive for positive values of *r*. (*r* must be positive as it is a physical area) Also, it must be a minimum, as all positive quadratics are U shaped (the only point where the gradient is zero is the minimum).

$$4\pi r - 2r^{-2} = 0$$

$$4\pi r = \frac{2}{r^2}$$

$$4\pi r^3 = 2$$

$$r^3 = \frac{1}{2\pi}$$

$$r = \sqrt[3]{\frac{1}{2\pi}}$$

by the same method, for any volume (*v*):

$$r = \sqrt[3]{\frac{v}{2\pi}}$$