

PAIR SUMS

How to form ten different sums out of five numbers? : 0, 2, 4, 4, 6, 8, 9, 11, 13, 15?

We are looking for five numbers which are named: a, b, c, d, e.

All possible sums of two numbers are therefore:

$$\begin{array}{cccc} a + b & b + c & c + d & d + e \\ a + c & b + d & c + e & \\ a + d & b + e & & \\ a + e & & & \end{array}$$

1. Because the sum of two numbers is 0, we conclude that the numbers are opposite numbers a and -a.
2. If we calculate all the sums we realize that each number is repeated four times in a sum. Therefore, these numbers are a, -a, c, d and e.

$$4 \cdot a + 4 \cdot (-a) + 4 \cdot c + 4 \cdot d + 4 \cdot e = 0 + 2 + 4 + 4 + 6 + 8 + 9 + 11 + 13 + 15$$

$$4 \cdot a + 4 \cdot (-a) + 4 \cdot c + 4 \cdot d + 4 \cdot e = 72$$

Because the numbers a and -a are opposite, the statement is valid for

$$4 \cdot a + 4 \cdot (-a) = 0$$

and:

$$4 \cdot (c + d + e) = 72$$

The equation is divided by four and result is:

$c + d + e = 18$

3. I reached the final result by systematic combining of all possibilities and excluding those which are not appropriate.

1. $c + d = 0$ $e = 18$ it's not possible because $18 > 15$
2. $c + d = 2$ $e = 16$ it's not possible because $16 > 15$
3. $c + d = 4$ $e = 14$

c	d	e	All possible sums	T/F
1	3	14	4, 15, 17	F
2	2	14	4, 16, 16	F

4. $c + d = 6$ $e = 12$

c	d	e	All possible sums	T/F
1	5	12	6, 13, 17	F
2	4	12	6, 14, 16	F
3	3	12	6, 15, 15	F

5. $c + d = 8$ $e = 10$

c	d	e	All possible sums	T/F
1	7	10	8, 11, 17	F
2	6	10	8, 12, 16	F
3	5	10	8, 13, 15	T
4	4	10	8, 14, 14	F

6. $c + d = 9$ $e = 9$

c	d	e	All possible sums	T/F
1	8	9	9, 10, 17	F
2	7	9	9, 11, 16	F
3	6	9	9, 12, 15	F
4	5	9	9, 13, 14	F

7. $c + d = 11$ $e = 7$

c	d	e	All possible sums	T/F
1	10	7	8, 11, 17	F
2	9	7	9, 11, 16	F
3	8	7	10, 11, 15	F
4	7	7	11, 11, 14	F
5	6	7	11, 12, 13	F

8. $c + d = 13$ $e = 5$

c	d	e	All possible sums	T/F
1	12	5	6, 13, 17	F
2	11	5	7, 13, 16	F
3	10	5	8, 13, 15	T
4	9	5	4, 13, 14	F
5	8	5	10, 13, 13	F
6	7	5	11, 12, 13	F

9. $c + d = 15$ $e = 3$

c	d	e	All possible sums	T/F
1	14	3	4, 15, 17	F
2	13	3	5, 15, 16	F
3	12	3	6, 15, 15	F (sum doesn't appear twice)
4	11	3	7, 14, 15	F
5	10	3	8, 13, 15	T
6	9	3	9, 12, 15	F
7	8	3	10, 11, 15	F

CONCLUSION: 3 out of five numbers which are only possible are 3, 5, 10.

4. I quickly conclude the opposite numbers must be numbers 1 and -1 .

Five unknown numbers are: $-1, 1, 3, 5, 10$.

LET'S CHECK: $1 + (-1) = 0$

$$1 + 3 = 4$$

$$1 + 5 = 6$$

$$1 + 10 = 11$$

$$-1 + 3 = 2$$

$$-1 + 5 = 4$$

$$-1 + 10 = 9$$

$$3 + 5 = 8$$

$$3 + 10 = 13$$

$$5 + 10 = 15$$