

Solution to Sextet

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$x+1/x=1$ (Note that x has no real solutions)

Let $A_1 = x+1/x=1$ and $A_n = x^n+1/x^n$.

We have $A_0=1+1=2$

For $A_2 = x^2+1/x^2$

Since $(x+1/x)^2 = x^2+2+1/x^2 = 1 \times 1 = 1$

Thus $A_2 = x^2+1/x^2 = 1-2 = -1 = A_1 - A_0$

For $A_3 = x^3+1/x^3$

Since $A_2 = x^2+1/x^2 = (x^2+1/x^2)(x+1/x) = x^3+x+1/x+1/x^3 = A_3 + A_1$

Therefore $A_3 = x^3+1/x^3 = A_2 - A_1 = -1 - 1 = -2$

In general,

$A_{n-1} = x^{n-1}+1/x^{n-1} = (x^{n-1}+1/x^{n-1})(x+1/x) = x^n+x^{n-2}+1/x^n+1/x^{n-2} = A_n + A_{n-2}$

Thus, $A_n = A_{n-1} - A_{n-2}$

From $A_0=2$ and $A_1=1$, we can generate the whole sequence of $\{A_n\}$ as follows:

2, 1, -1, -2, -1, 1, **2, 1, -1, -2, -1, ...**

We can see that the sequence is a repeating pattern of 2, 1, -1, -2, -1, 1 for successive values of n , with a period of 6.