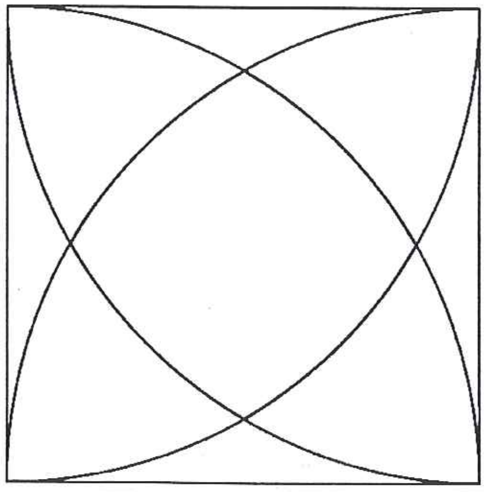
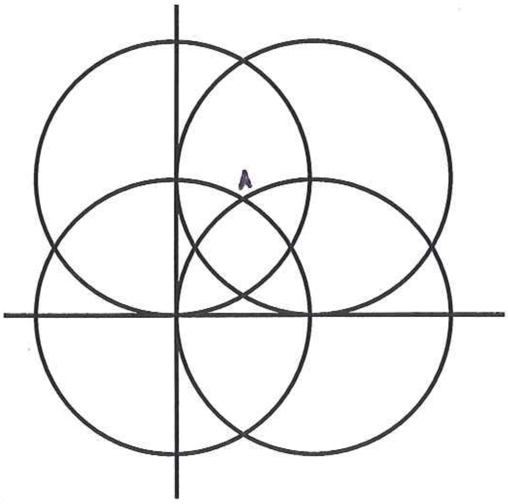


If the diagram is drawn on a graph, then the circles can be given the equations:

$x^2+y^2=1$ $\qquad$ $x^2+(y-1)^2=1$

$(x-1)^2+y^2=1$ $\qquad$ $(x-1)^2+(y-1)^2=1$



Point A is the positive point where '$x^2+y^2=1$' and '$(x-1)^2+y^2=1$' cross. This can be found using simultaneous equations.

$x^2+y^2=1$

$x^2=1-y^2$

$x=\sqrt{1-y^2}$ $\rightarrow$ substitute this into '$(x-1)^2+y^2=1$'

$\left(\sqrt{1-y^2}-1\right)^2+y^2=1$

$1-y^2+1-2\sqrt{1-y^2}+y^2=1$

$-2\sqrt{1-y^2}=1$

$\sqrt{1-y^2}=-\frac{1}{2}$

$1-y^2=\frac{1}{4}$

$y^2=\frac{3}{4}$

$y=\pm\frac{\sqrt{3}}{2}$

We want the positive point where the circles cross so we know the y co-ordinate is $\frac{\sqrt{3}}{2}$.
We then put this value back into the equation '$x^2+y^2=1$':

$x^2+\left(\frac{\sqrt{3}}{2}\right)^2=1$

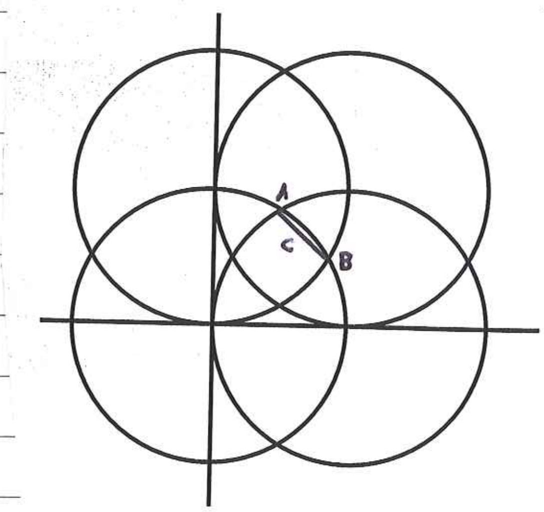
$x^2+\frac{3}{4}=1$

$x^2=\frac{1}{4}$

$x=\pm\frac{1}{2}$

We want the positive point so the x co-ordinate is $\frac{1}{2}$.

We now know that point A has co-ordinates $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, using the same process we can find the co-ordinates of B.



$x^2+y^2=1 \qquad x^2+(y-1)^2=1$

$x^2 = 1-y^2$

$x = \sqrt{1-y^2}$

$\left(\sqrt{1-y^2}\right)^2 + (y-1)^2 = 1$

$1-y^2+y^2+1-2y=1$

$2-2y=1$

$2y=1$

$y=\frac{1}{2}$

$x^2+\left(\frac{1}{2}\right)^2=1$

$x^2+\frac{1}{4}=1$

$x^2=\frac{3}{4}$

$x=\frac{\sqrt{3}}{2}$

$\therefore$ Point B has co-ordinates of $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

We can find the distance between A and B using Pythagorus.

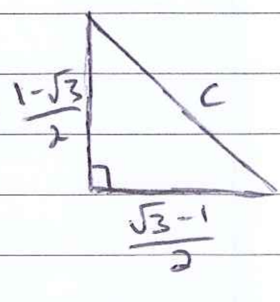
$\frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

$a^2+b^2=c^2$

$\frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

$\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{1-\sqrt{3}}{2}\right)^2 = c^2$

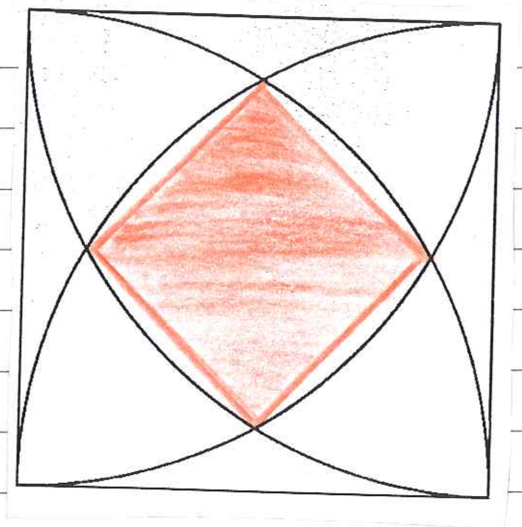
$\frac{4-2\sqrt{3}}{4} + \frac{4-2\sqrt{3}}{4} = c^2$



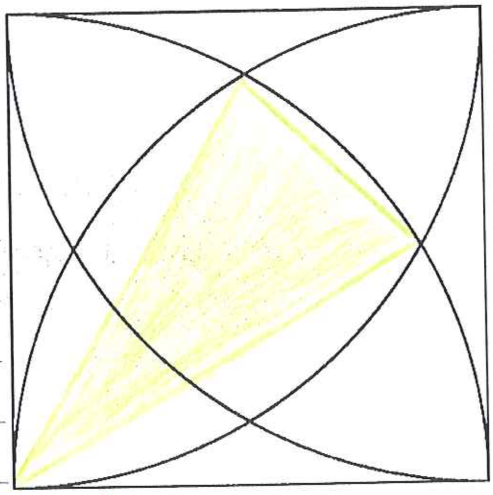
$c^2 = 2-\sqrt{3}$

$c = \sqrt{2-\sqrt{3}}$

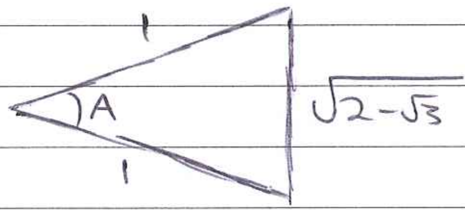
C is one of the sides of the red square so we can square it to find the red square's area.



$$\text{Red Square Area} = \left(\sqrt{2-\sqrt{3}}\right)^2$$
$$= 2-\sqrt{3}$$



Using the values we have already found and the knowledge that the radius of all the circles is 1, we can find all the sides of the green triangle.



We can use the cosine rule to find A.

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos A = \frac{1+1-\left(\sqrt{2-\sqrt{3}}\right)^2}{2}$$

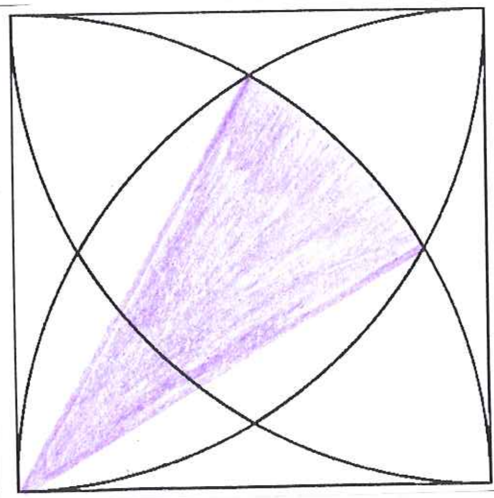
$$\cos A = \frac{2-2+\sqrt{3}}{2}$$

$$\cos A = \frac{\sqrt{3}}{2}$$

$$A = 30^\circ$$

$$\frac{30^\circ}{360^\circ} = \frac{1}{12}$$

$\therefore$ The sector containing the green triangle is $\frac{1}{12}$ of the circle.



The circle has a radius of 1.

Area of the circle $= \pi r^2$

$= \pi \times 1$

$= \pi$

$\therefore \frac{\pi}{12} =$ Area of the purple sector

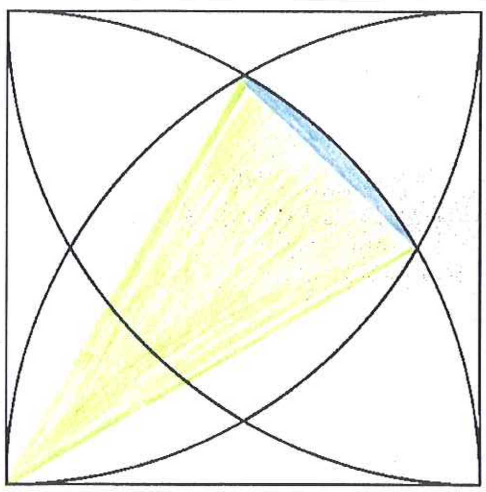
Area of the purple sector $-$ Area of the green triangle $=$ Area of the blue segment.

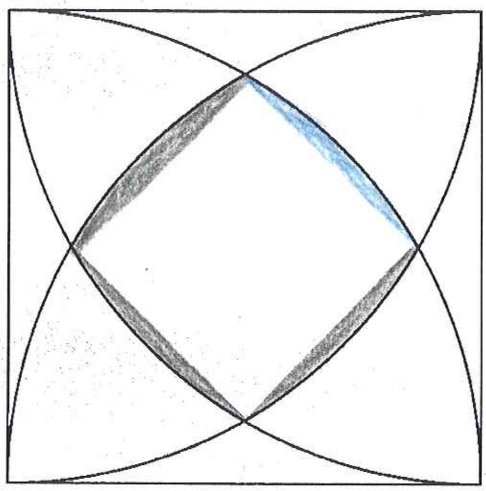
$\frac{\pi}{12} - \left(\frac{1}{2} bc \sin A\right) =$ Area of the blue segment.

$\frac{\pi}{12} - \left(\frac{1}{2} \times 1 \times 1 \times \sin 30\right) =$ Area of the blue segment

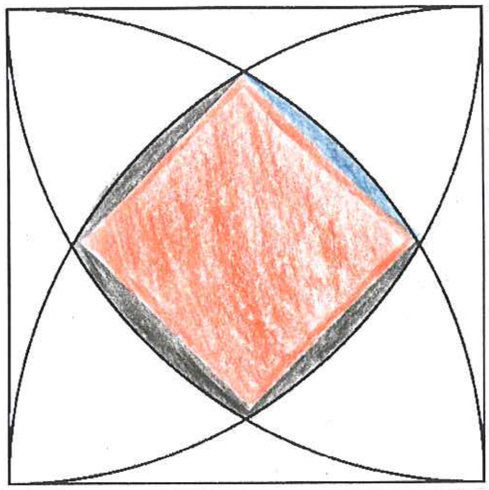
$\frac{\pi}{12} - \frac{1}{4} = \frac{\pi - 3}{12} =$ Area of the blue segment.

There are three more identical segments and the combined area of the four $= \frac{\pi - 3}{3}$





The area of the red square plus these segments equals the area of the shaded shape.



$2-\sqrt{3} + \frac{\pi - 3}{3} = $ Shaded Area

Shaded Area $= \frac{\pi - 3}{3} + \frac{6 - 3\sqrt{3}}{3}$

Shaded Area $= \frac{\pi + 3 - 3\sqrt{3}}{3}$

Shaded Area $= 0.315$ (To 3 significant figures)