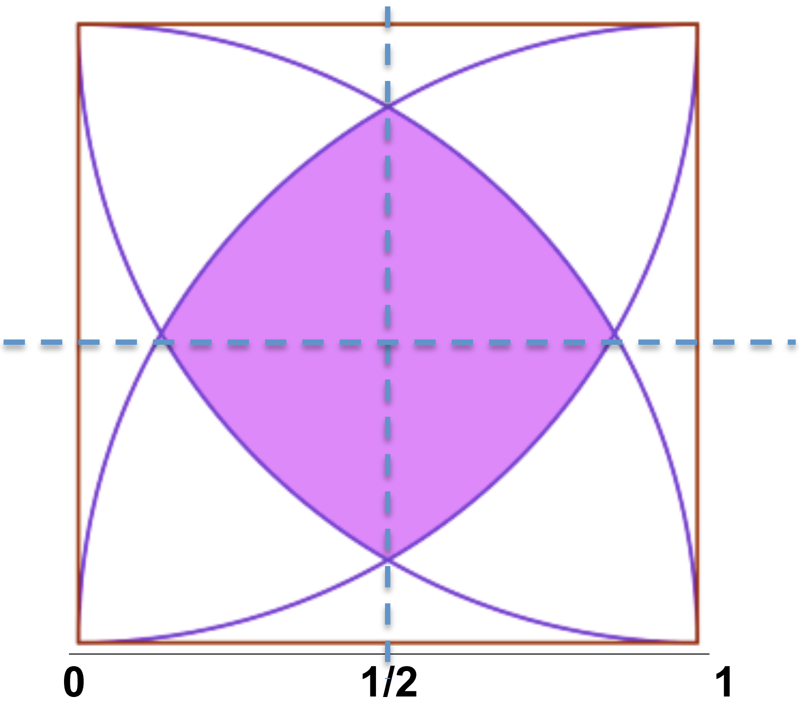
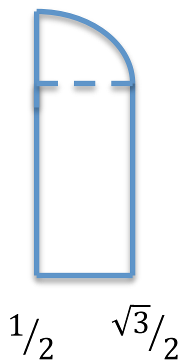
Curved Square



Due to the symmetrical nature of the shape, a good approach could be to find a quarter of the area, then multiplying it by 4. Here, we shall consider the top-right quarter of the shape.
Firstly, we need to know the bounds of integration. Obviously, the lower limit would be ½, so this has been added to the diagram
For the upper bound, we want the x coordinate of the rightmost vertex of the shape. At this point, $y = ½$. It can also be noted that the circle with equation $x^2 + y^2 = 1$ passes through this point. Therefore, $x^2 = {}^3/_4$, so $x = {}^{\sqrt{3}}/_2$.

Now, by integration, we are finding the following area, with the desired area being above the dotted line:



For the integration, we require an explicit formula in the form $y = f(x)$. For the circle $x^2 + y^2 = 1$, a form (valid for $y > 0$) is $y = \sqrt{1 - x^2}$

We therefore need to compute the following integral:

$$\int_{1/2}^{\sqrt{3}/2} \sqrt{1-x^2}\, dx$$

By substituting $x = \cos(t)\,, \frac{dx}{dt} = -\sin\,(t)$, and after changing the limits, we are left with:

$$-\int_{\pi/3}^{\pi/6} (\sin t)^2 \;\; dt$$

By noting that – $(\sin(t))^2 = {}^1\!/_2\,(\cos(2t) - 1)$, we can compute the integral to be:

$${}^1\!/_2\left[{}^1\!/_2 \sin(2t) - t\right]_{\pi/3}^{\pi/6}$$

$$= {}^1\!/_2\left\{\left({}^{\sqrt{3}}\!/_4 - {}^{\pi}\!/_6\right) - \left({}^{\sqrt{3}}\!/_4 - {}^{\pi}\!/_3\right)\right\}$$

$$= {}^{\pi}\!/_{12}$$

Now we need to subtract the area of the rectangle below the dotted line in the above drawing. This area is simply given by $\frac{\sqrt{3}-1}{2} * \frac{1}{2} = \frac{\sqrt{3}-1}{4}$

So we have worked out the area of 1 quarter of the desired shape to be:

$$\frac{\pi}{12} - \frac{\sqrt{3}-1}{4}$$

The area of the desired shape is therefore $\mathbf{1 + \frac{\pi}{3} - \sqrt{3}}$