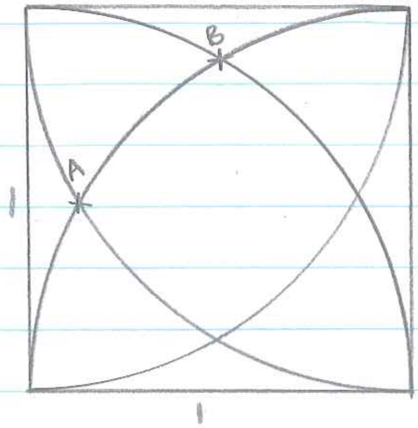
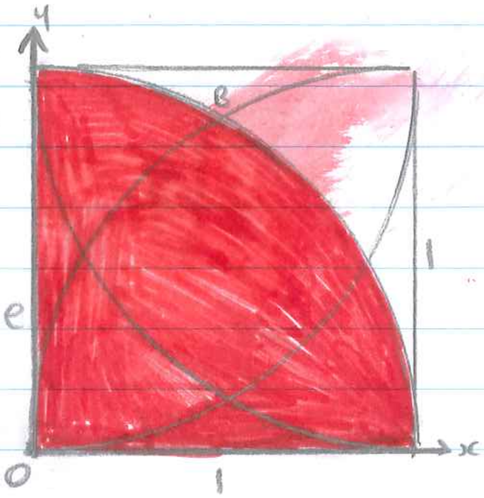
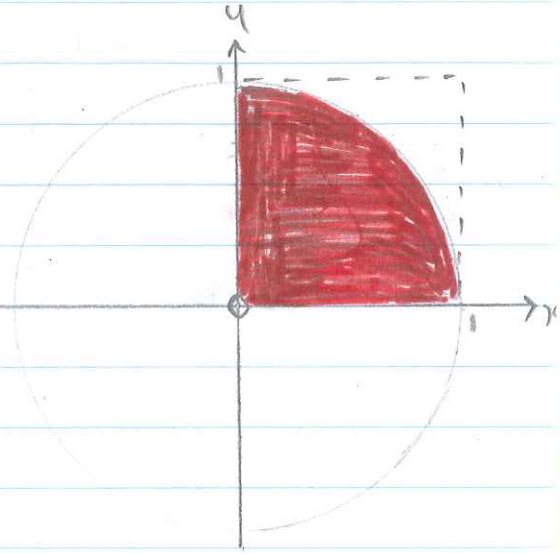
BRANDON O'CONNELL



• in order to find the area of the largest square, we need to find the length AB. to do this I will treat the diagram as an image on an axis and will use coordinates to find AB.

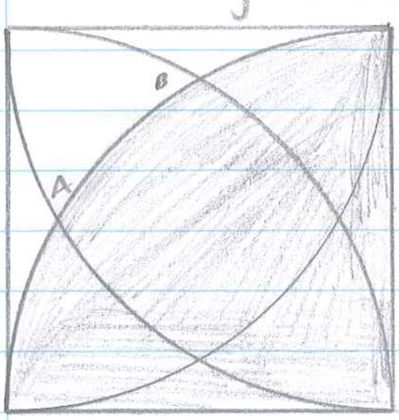
• treat the bottom left corner of the square (centre of sector shown) as the origin.



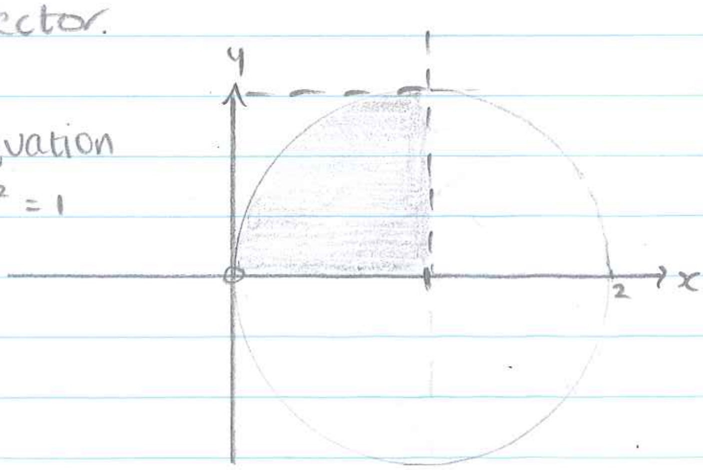


• the equation of the circular graph shown is $x^2+y^2=1$

• Now using a different sector.



• with equation $(x-1)^2+y^2=1$



• using these two equations we can use simultaneous equations to find the coordinates of B

① $x^2 + y^2 = 1$
② $(x-1)^2 + y^2 = 1$ } Two equations

rearrange ①

$$y^2 = 1 - x^2$$

and substitute into ②

$$(x-1)^2 + 1 - x^2 = 1$$

$$(x-1)^2 - x^2 = 0$$

Expand and simplify

$$x^2 - 2x + 1 - x^2 = 0$$
$$-2x + 1 = 0$$

$$0 = 2x - 1$$
$$2x = 1$$
$$x = \frac{1}{2}$$

if $x = \frac{1}{2}$

using $x^2 + y^2 = 1$

$$\left(\frac{1}{2}\right)^2 + y^2 = 1$$

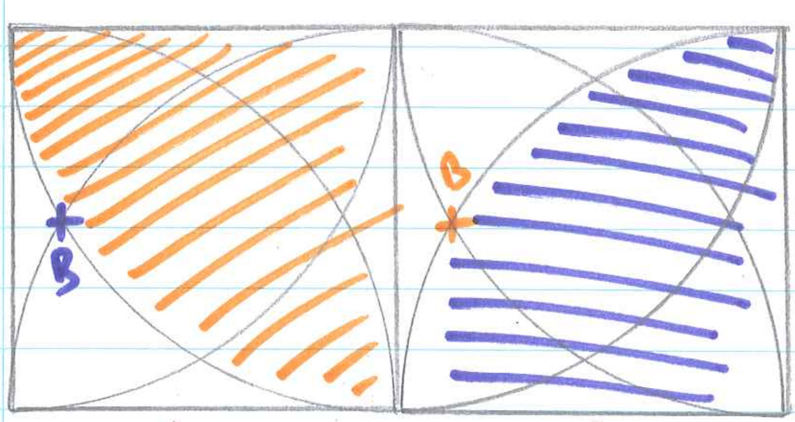
$$\frac{1}{4} + y^2 = 1$$

$$y^2 = 1 - \frac{1}{4}$$

$$y^2 = \frac{3}{4} \qquad y = \frac{\sqrt{3}}{2}$$

$\therefore$ coordinate of B is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

- now, we do the same process as before, but with the other two sectors that will give us coordinate A.



The equation of the first one is $(x-1)^2+(y-1)^2=1$

The equation of the second is one we used before. $(x-1)^2 + y^2 = 1$

(using same process as before)

- Once again we use simultaneous equations to find coordinate B

① $(x-1)^2 + y^2 = 1$
② $(x-1)^2 + (y-1)^2 = 1$

rearrange ① $(x-1)^2 = 1 - y^2$

then substitute

$1 - y^2 + (y-1)^2 = 1$

and expand.

$\cancel{1} - y^2 + (y-1)^2 = \cancel{1}$

$-y^2 + (y-1)^2 = 0$

$-y^2 + y^2 - 2y + 1 = 0$

· Simplify

$-2y + 1 = 0$

$0 = 2y - 1$

$1 = 2y$

$\frac{1}{2} = y$

let $y = \frac{1}{2}$ in $(x-1)^2 + y^2 = 1$

$(x-1)^2 + (\frac{1}{2})^2 = 1$

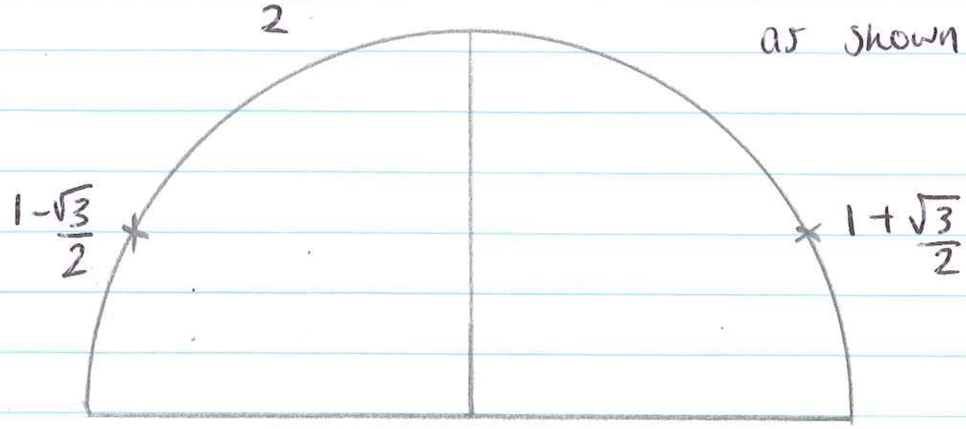
$(x-1)^2 + \frac{1}{4} = 1$

$(x-1)^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\sqrt{(x-1)^2} = \frac{\pm\sqrt{3}}{\sqrt{4}} = \frac{\pm\sqrt{3}}{2}$

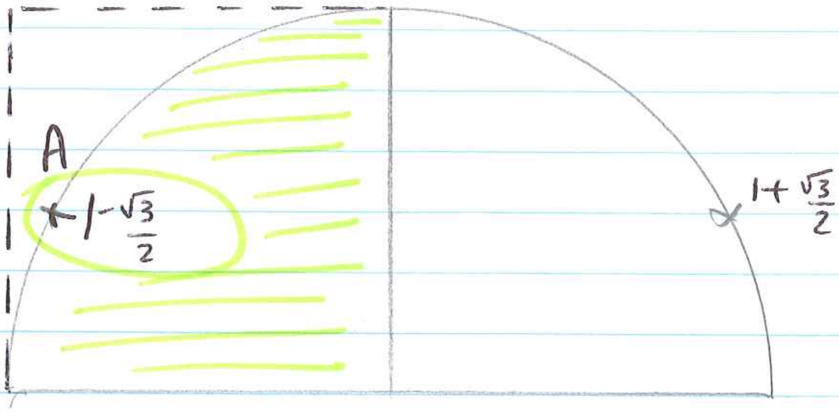
$x - 1 = \frac{\pm\sqrt{3}}{2}$

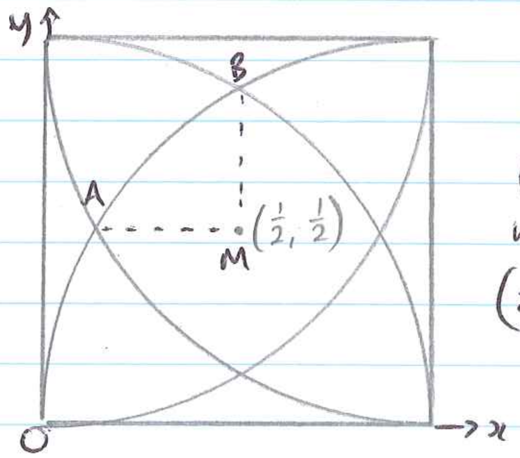
$x = 1 \pm \frac{\sqrt{3}}{2}$ · We have two coordinates as shown



· looking back at the first diagram, we need the one to the left, that is the same as the sector connected to A.

$\therefore$ coordinate A is $\left(1-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$





By symmetry and by putting perpendiculars in from A and B, we get the centre of square $(\frac{1}{2}, \frac{1}{2})$ M

- Do $\frac{1}{2}$ minus the x coordinate of A to work out length from A to M.

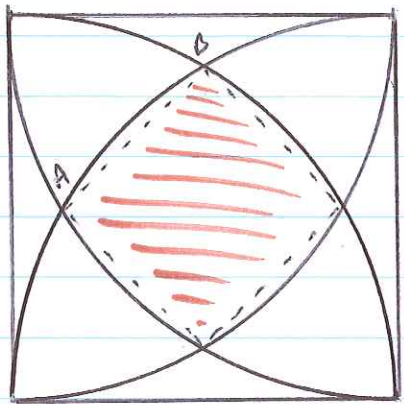
$$\frac{1}{2} - \left(1 - \frac{\sqrt{3}}{2}\right) = AM = \frac{1}{2} - 1 + \frac{\sqrt{3}}{2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3} - 1}{2} = AM$$

· Using pythagoras we can find AB^2 and so the area of the square. We can also determine the length AB

$$\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}-1}{2}\right)^2 = AB^2 = 2\left(\frac{\sqrt{3}-1}{2}\right)^2$$



$$AB^2 = \frac{(\sqrt{3}-1)^2}{2}$$

$$AB^2 = \frac{3-2\sqrt{3}+1}{2}$$

$$AB^2 = \frac{4-2\sqrt{3}}{2}$$

AB^2 = Area of square $= \underline{\underline{2-\sqrt{3}}}$

Now we can find the length of AB using

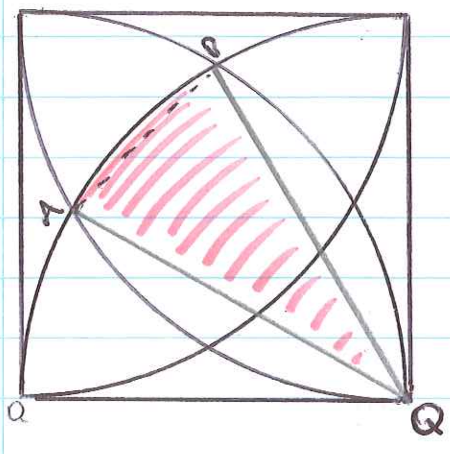
$$\frac{(\sqrt{3}-1)^2}{2}.$$

$$\sqrt{AB^2} = \sqrt{\frac{(\sqrt{3}-1)^2}{2}} = \frac{\sqrt{3}-1}{\sqrt{2}}$$

rationalise.

$$\frac{\sqrt{3}-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{2}$$

$\frac{\sqrt{6}-\sqrt{2}}{2} = AB$ ← Also equals other sides as its a square and symmetry rules apply.



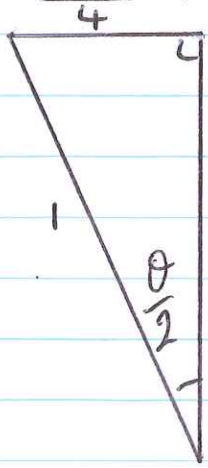
- Now create a sector that comes from AB and drops to the sector centre, Q.
- We shall also call $\angle AQB = \theta$

- To find the remaining area in the original shaded area we need the square area plus the out curved regions. Segments.

- In order to find area of segment we need θ.

- Create a triangle that is bisector of AB and θ and has a side 1 and $\left(\frac{\sqrt{6}-\sqrt{2}}{2}\right) \div 2$.

$\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{1}{2}AB$



- Work in Radians.
- Use $\sin x = \frac{O}{H}$

$$\sin\left(\frac{\theta}{2}\right) = \frac{\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)}{1} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\frac{\theta}{2} = \sin^{-1}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

$$\theta = 2\sin^{-1}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

$$\theta = \frac{\pi}{6}$$

• Using the area of a segment formula $\frac{1}{2}r^2(\theta - \sin\theta)$

we can find the area of original shaded region.

Sub r for arc radius 1
and θ for $\frac{\pi}{6}$

$$\frac{1}{2}1^2\left(\frac{\pi}{6} - \sin\left(\frac{\pi}{6}\right)\right) = \text{area of segment.}$$

due to symmetry and that $\frac{\pi}{6} = 12\%$ of full 2π (circle)

the four segments are equal.

$$\frac{1}{2}\left[\frac{\pi}{6} - \sin\left(\frac{\pi}{6}\right)\right] \times 4$$

$$= 2\left[\frac{\pi}{6} - \sin\left(\frac{\pi}{6}\right)\right]$$

then we must add this area to the square area.

$$2\left[\frac{\pi}{6} - \sin\frac{\pi}{6}\right] + 2 - \sqrt{3} = \text{Area of Shaded}$$

$$= \left(\frac{\pi}{3} - 1\right) + 2 - \sqrt{3}$$

$$= \frac{\pi}{3} + 1 - \sqrt{3}$$

$$= 0.3151674336$$