ADDING ODD NUMBERS

- $1 = 1$
  $4 = 1 + 3$
  $9 = 1 + 3 + 5$
  $16 = 1 + 3 + 5 + 7$
  $25 = 1 + 3 + 5 + 7 + 9$
  $36 = 1 + 3 + 5 + 7 + 9 + 11$
  $49 = 1 + 3 + 5 + 7 + 9 + 11 + 13$
  $64 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$
  $81 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$
  $100 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

We see that sum of these odd numbers is always a perfect square. To be more specific, sum of first n odd numbers is n^2. Short proof by induction below:

- So, sum of first 100 numbers according to observation should be $(100)^2 = 10{,}000$

- Base case: $n=1$, sum of 1 odd number $= 1 = 1^2$. True for $n=1$.

Assuming true for $n=k$, $1+3+5+\ldots(2k-1) = k^2$

Consider $n=k+1$, so $1+3+5+\ldots+(2k-1)+(2k+1)$ should be $(k+1)^2$

but, $1+3+5+\ldots+(2k-1) = k^2$

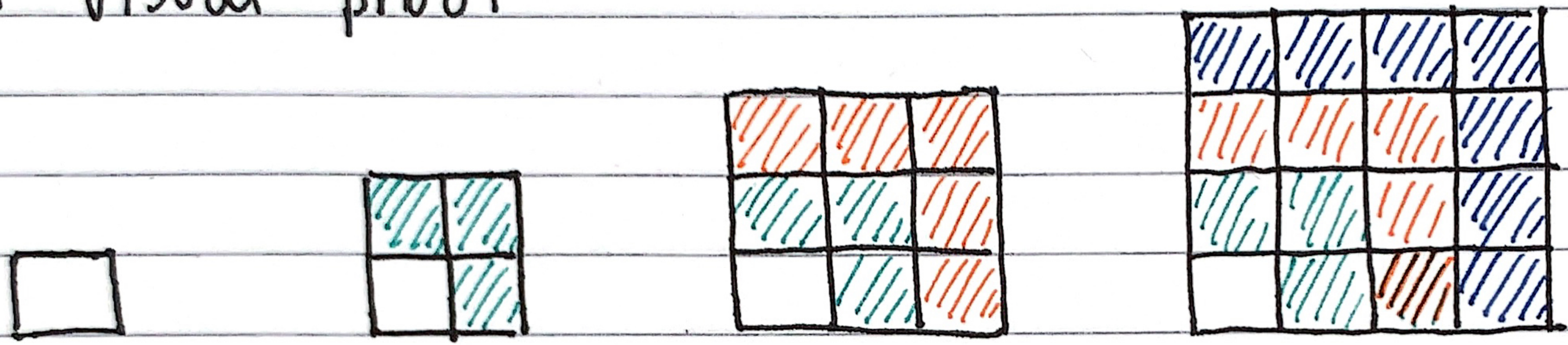
so, $(k^2) + 2k+1$ should be $(k+1)^2$

$k^2 + 2k + 1$ should be $(k+1)^2$

And indeed $k^2 + 2k + 1 = (k+1)^2$

Since true for $n=1$, $n=k$, $n=k+1$, it is true for all int. $n \geq 1$

A visual proof



$1 \qquad 1+3 \qquad 1+3+5 \qquad 1+3+5+7$

$=1 \qquad =4 \qquad =9 \qquad =16$

Each step after moving from $(1)^2$ to $(2)^2$ to $(3)^2$ to $(4)^2$, odd numbers are added.