# Curve Fitter

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## Part 1

The general form of a cubic is $y = ax^3 + bx^2 + cx + d$

We are given three co-ordinates that satisfy this cubic. By direct substitution, we obtain a few equations.

$0 = a(0)^3 + b(0)^2 + c(0) + d$
$0 = d$

$2 = a(1)^3 + b(1)^2 + c(1) + d$
$2 = a + b + c + d$
$2 = a + b + c \quad [\because d = 0]$

$1 = a(2)^3 + b(2)^2 + c(2) + d$
$1 = 8a + 4b + 2c + d$
$1 = 8a + 4b + 2c \quad [\because d = 0]$

We have three variables but only two conditions, hence we can eliminate or represent other two variables in the form of one variable, say 'a'

$8a + 4b + 2c = 1$
$a + b + c = 2 \quad \implies \quad 2a + 2b + 2c = 4$

Subtract the two equations

$$6a + 2b = -3 \implies b = \frac{-3 - 6a}{2}$$

Substitute in any equation to get $c$

$$a + b + c = 2 \quad \implies \quad a + \frac{-3 - 6a}{2} + c = 2 \quad \implies \quad c = \frac{7 + 4a}{2}$$

Hence the function is:

$$y = ax^3 + \left(\frac{-3 - 6a}{2}\right)x^2 + \left(\frac{7 + 4a}{2}\right)x + 0$$

## Part 2 (a)

We are given an additional condition that the turning point of this cubic is at (1,2). At turning points, the derivative i.e. the slope is 0

Hence $y' = 0$ at $x = 1$

Applying the power rule, we get the derivative of the function as follows:

$$y' = 3ax^2 + 2\left(\frac{-3-6a}{2}\right)x + 1\left(\frac{7+4a}{2}\right)$$

Substitue $y' = 0$ and $x = 1$

$$0 = 3a(1)^2 + (-3-6a)(1) + \left(\frac{7+4a}{2}\right)$$

Simplify and solve for $a$

$$0 = (3a - 6a + 2a) + (-3 + 3\frac{1}{2}) \quad \Longrightarrow \quad 0 = -a + \frac{1}{2} \quad \Longrightarrow \quad a = \frac{1}{2}$$

## Part 2 (b)

We are given an additional condition that the turning point of this cubic is at (2,1). At turning points, the derivative i.e. the slope is 0. Hence $y' = 0$ at $x = 2$

Applying the power rule, we get the derivative of the function as follows:

$$y' = 3ax^2 + 2\left(\frac{-3-6a}{2}\right)x + 1\left(\frac{7+4a}{2}\right)$$

Substitue $y' = 0$ and $x = 2$

$$0 = 3a(2)^2 + (-3-6a)(2) + \left(\frac{7+4a}{2}\right)$$

Simplify and solve for $a$

$$0 = (12a - 12a + 2a) + (-6 + 3\frac{1}{2}) \quad \Longrightarrow \quad 0 = 2a - 2\frac{1}{2} \quad \Longrightarrow \quad a = 1\frac{1}{4}$$

## Part 3

For the function to be able to be a turning point at (1,2) and (2,1) there can only exist one value of $a$. But here there are distinct values $a$ for which each of these conditions are true.

If $a = \frac{1}{2}$, then only (1,2) is a turning point. If $a = 1\frac{1}{4}$, then only (2,1) is a turning point.
Hence there is no such value of $a$ for which the turning points are (1,2) AND (2,1)