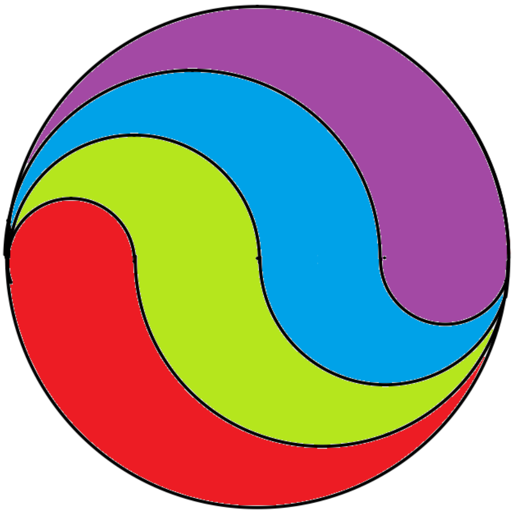

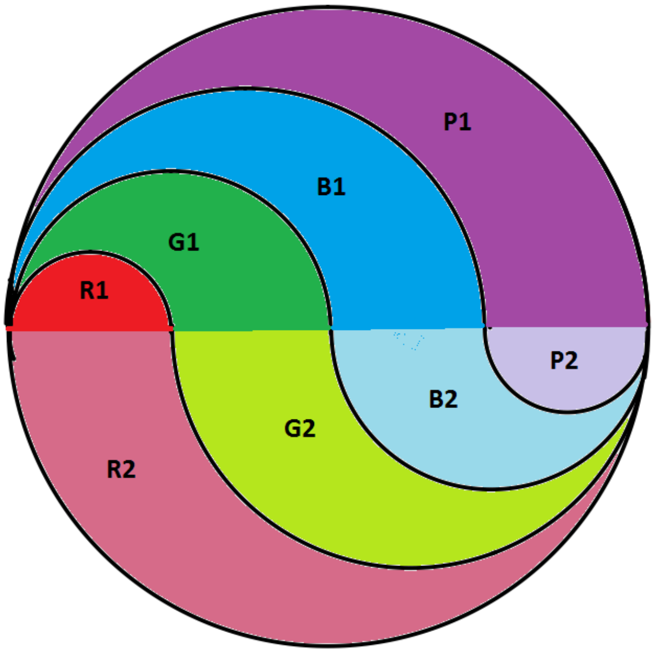

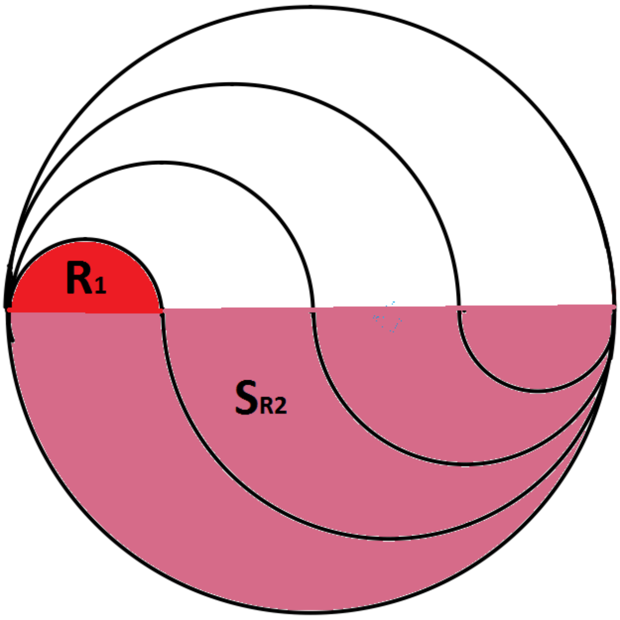

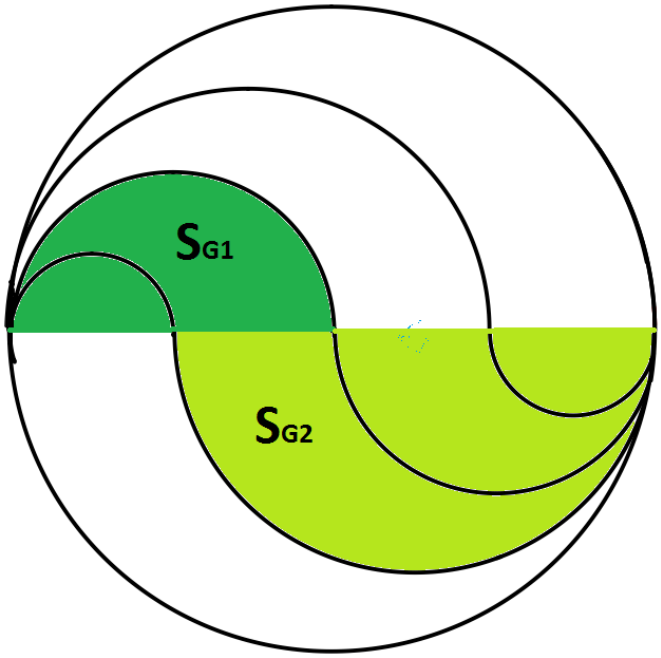

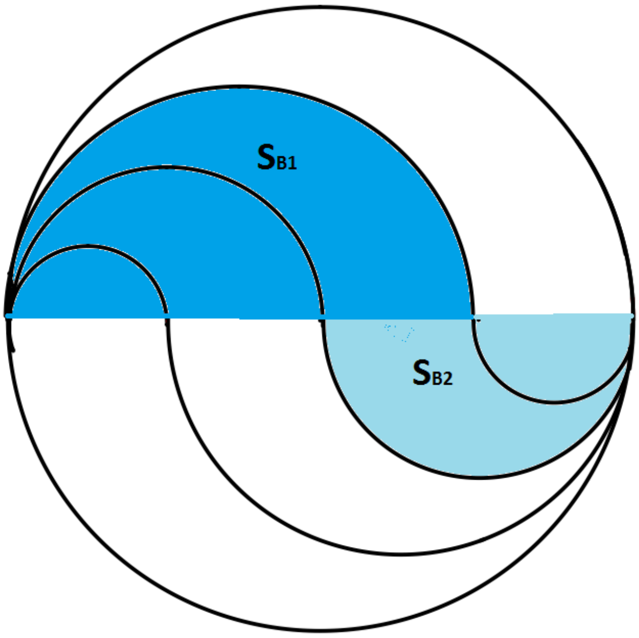

Let each of the labels define the area inside that shaded region.

And

Let the notation "Rad(R_1)" denote the radius of the section with area R_1.

$\text{Rad}(R_1) = \text{Rad}(P_2) = x$

$\text{Rad}(S_{G1}) = \text{Rad}(S_{B2}) = 2x$

$\text{Rad}(S_{B1}) = \text{Rad}(S_{G2}) = 3x$

$\text{Rad}(S_{R2}) = 4x$

I have hours of fun working through these NRich problems, and this one has certainly taken some time! This may not be an ideal solution and proof, but is a notation I am comfortable with using through my own experiences.

Throughout the solution there are points for the reader to reflect on the progress being made and check understanding, or to work through part of the problem.

On the previous page, I have presented a number of pictures, to show the notation that will be used throughout this solution. I will first find the proportion each colour takes up of a circle when we use 4 colours, and go on to make a conjecture and proof of a general case.

CONSTRUCTION

The working here assumes the reader has worked out how these circles are constructed. As an example perhaps try:

- Drawing a circle radius 4.
- Use a ruler to make a mark a point directly 3cm left of the centre point.
- From this draw a semi-circle of radius 1 (this is my R_1)
- To form R_2, find the mid-point of the horizontal line from the RHS of R_1 to the circumference. Mark this point.
- Placing the tip of the compass on this point and the pencil on the bottom right R_1 draw the line which forms the region R_2
- To form G_1 (in my picture), measure double the radius for the first semi-circle (2cm) with the compass, place the point of the compass on the bottom right of R_1 and draw the semi-circle.
- Repeating these steps is one way to form this pattern.

There are of course different ways to draw these patterns, for example, drawing lots of different semi-circles and rubbing out the lines (you may find this easier), but the above was my preferred method.

It is definitely worth drawing a couple of these patterns to get a feel for how the shapes are formed.

SOLUTION WITH 4 COLOURS

Suggestions:

- Before attempting to follow thus general solution, try working through a numerical solution. Maybe for one of your drawings?

- If you have worked through numerical solutions, but still find the general example seems too tricky to start with, try substituting a number for x. Does it match with your numerical workings?

Let x be the radius of the region with area R_1.

$$Area\ of\ Whole\ Circle, \qquad A = (\ ????\ )^2\pi = ???$$

Therefore,

$$S_{R2} = 8\pi x^2$$

Also:

$$R_1 = P_2 = \frac{\pi x^2}{2}$$

$$S_{G2} = S_{B1} = \frac{(3x)^2\pi}{2} = \frac{9x^2\pi}{2}$$

$$S_{B2} = S_{G1} = ???$$

Try the last one for yourself; use the pictures to help your understanding!

What is the benefit of finding all of these equations at this point?

Step 1: Find Area of Red Section, R

$$\boldsymbol{R = R_1 + R_2}$$

$$R_1 = \frac{\pi x^2}{2}$$

$$R_2 = S_{R2} - S_{G2} = 8\pi x^2 - \frac{9\pi x^2}{2} = \frac{7\pi x^2}{2}$$

$$R = \frac{\pi x^2}{2} + \frac{7\pi x^2}{2} = \frac{8\pi x^2}{2} = 4\pi x^2$$

Step 2: Find area of green section, G

$$\boldsymbol{G = G_1 + G_2}$$

$$G_1 = S_{G1} - R_1 = 2x^2\pi - \frac{\pi x^2}{2} = \frac{3\pi x^2}{2}$$

$$G_2 = S_{G2} - S_{B2} = \frac{9x^2\pi}{2} - 2x^2\pi = \frac{5\pi x^2}{2}$$

$$G = \frac{3\pi x^2}{2} + \frac{5\pi x^2}{2} = \frac{8\pi\, x^2}{2} = 4\pi x^2$$

Step 2: Find area of blue section, B

$$\boldsymbol{B = B_1 + B_2}$$

Left as an exercise to the reader…

Step 3: Find area of purple section, P

$$P = P_1 + P_2$$

We could find this using the same method as previously, but is there an easier way, in which I can use reasoning to simply state the area, P?

How can we use the information we have at this point to check our areas are correct (most likely)?

Since, every colour has the same area and there are 4 different colours we can deduce that the proportion each colour takes up of the whole circle is ¼ .

I have used reasoning here to deduce the proportion is 1/4 ; what calculation could be performed instead?

Conjecture:

If we split any circle using this technique in to *n* different colours then the proportion each will take up will be **(Insert idea here).**

Proof:

I will adopt the following notation:

- Let the area of each separate colour be denoted by $C_0, C_1, C_2, \ldots, C_{n-1}$
- $C_{i,1}$ will represent the 'first-part' of colour i, *i.e. the parts being called R_1, G_1, B_1 and P_1 in the previous example*
- $C_{i,2}$ will represent the 'other-part' of colour, i, *i.e. the parts in the previous example denoted by R_2, G_2, B_2 and P_2.*
- Let, $S_{C_{i,1}} = \sum_{r=0}^{i} C_{r,1}$ (Previously labelled, as S_{B1}, S_{G1}, ….)
- Let, $S_{C_{i,2}} = \sum_{r=i}^{n-1} C_{r,2}$ (Previously labelled, as S_{B2}, S_{G2}, ….)

Now, for i= 0, 1, 2, . . . , (n -1)

Could i have started from 1? 2? 3? What difference would it make?

$$\boldsymbol{C_i = C_{i,1} + C_{i,2}}$$

$C_{i,1} = S_{C_{i,1}} - S_{C_{(i-1),1}}$ Define: $S_{C_{-1,1}} = 0$

$C_{i.2} = S_{C_{i,2}} - S_{C_{i+1,2}}$ Define: $S_{C_{n,1}} = 0$

Why have I included the extra 2 definitions?

Therefore,

$$C_i = S_{C_{i,1}} - S_{C_{(i-1),1}} + S_{C_{i,2}} - S_{C_{i+1,2}}$$

With a little thought (refer to pictures above), it can be shown that:

$$S_{C_{i,1}} = \frac{\pi x^2}{2}(i+1)^2$$

It follows that:

$$S_{C_{(i-1),1}} = \frac{\pi x^2}{2} i^2$$

How did I get from the expression for $S_{C_{i,1}}$ to the expression for $S_{C_{(i-1),1}}$?

Similarly, it can be shown that:

$$S_{C_{i,2}} = \frac{\pi x^2}{2}(n-i)^2$$

(Again, refer to pictures, to get a sense for how I got this)

And so,

$$S_{C_{i+1,2}} = ?????$$

How can we now find an equation for the area of each colour? Do we need just 1 general equation or a number of different equations?

HINT: It will really help to simplify your equation(s) as much as possible!

Then, h**ow can we use the information we have to show what proportion each colour takes up? Perhaps there is another thing we need… (Think about the suggestion in the example).**

Is this the last step of the proof? How do you know?