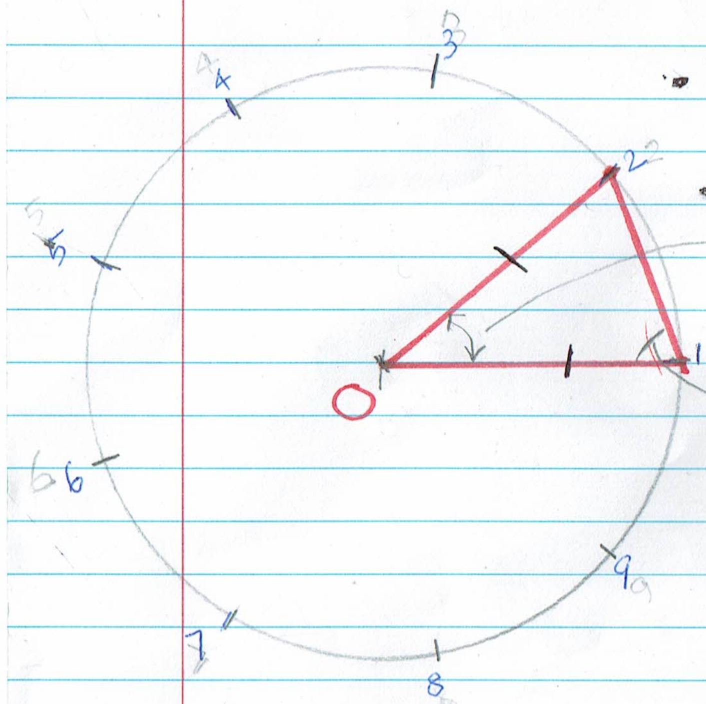
Cyclic Quadrilaterals

Explanation – 9 DOTS ON CIRCLE AS SAMPLE.



- Joined between two dots is
One sector $= 9 \div 1 = 9$ triangles at most

$360^\circ \div 9 = 40^\circ$

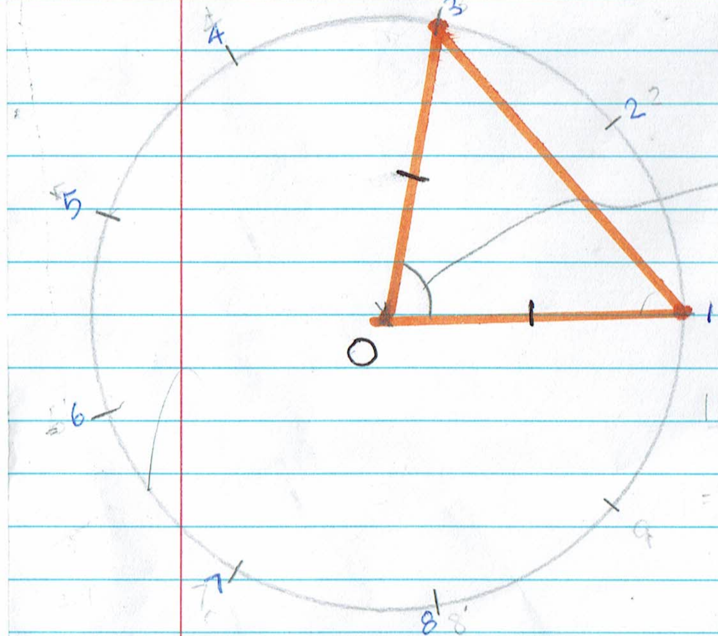
$\triangle 102$ is an isosceles triangle.

$\because \overline{O1} = \overline{O1} =$ radius

$\therefore \angle O12 = \frac{180^\circ - 40^\circ}{2}$

$= 70^\circ$

$= \angle O21$



2 sectors form 1 triangle

$\therefore \frac{9}{2} = 4R1$ $\therefore$ Only 4 triangles at any time

$\angle 103 = 2 \times 40^\circ$

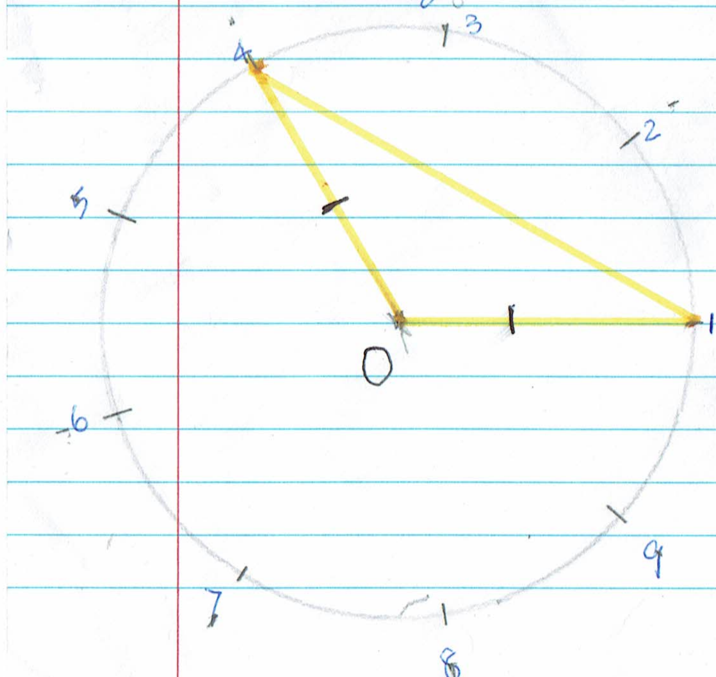
$= 80^\circ$

$\triangle 103$ is an isosceles triangle

$\angle O13 = \angle O31$

$= \frac{180 - 80}{2}$

$= 50^\circ$



3 sectors form 1 triangle

$\therefore \frac{9}{3} = 3R0$ $\therefore$ 3 triangles at most

$\angle 104 = 3 \times 40^\circ$

$= 120^\circ$

$\angle O14 = \frac{180 - 120}{2}$

$= 30^\circ$

$\angle O41 = \angle O14 = 30^\circ$

4 sectors ~~form~~ 1 triangle

$\frac{9}{4} = 2R1$ $\therefore$ Forms 2 triangles at anytime

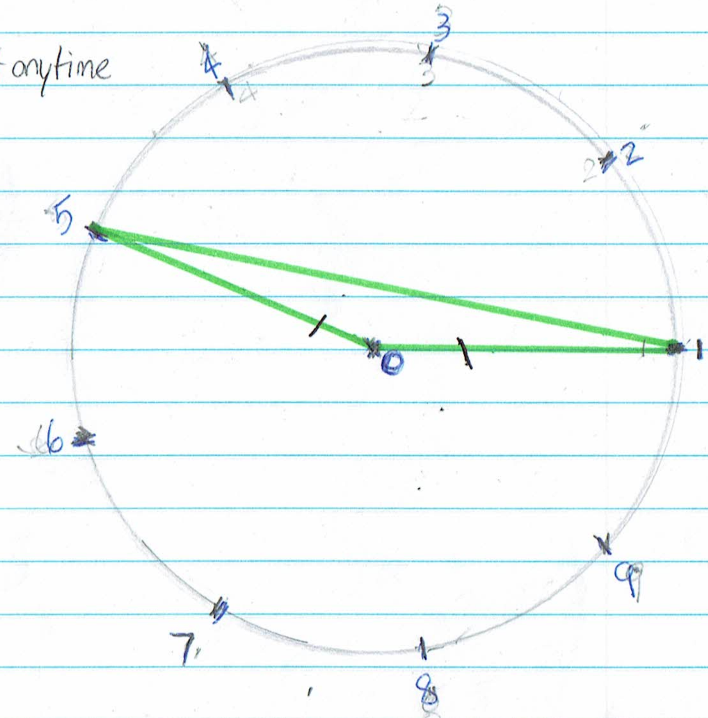
$\angle 1O5 = 4 \times 40^\circ$

$= 160^\circ$

$\angle O15 = \frac{180 - 160}{2}$

$= 10^\circ$

$\angle O15 = \angle O51 = 10^\circ$



This is a quadrilateral, as shown

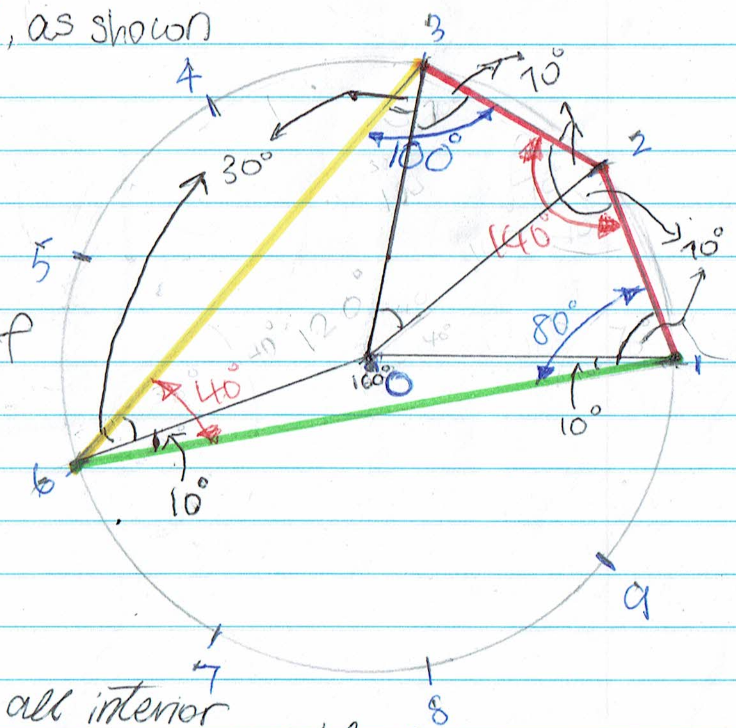
$140^\circ + 40^\circ = 180^\circ$

$\angle 123 + \angle 163 = 180^\circ$

$\angle 236 + \angle 216 = 100^\circ + 80^\circ$

$= 180^\circ$

$\therefore$ The opposite angles of a quadrilateral always add up to 180°



$180^\circ + 180^\circ = 360^\circ$ Sum of all interior angles of the quadrilateral

Ci Hui Minh Ngoc Ong, Age 11, Grade 7
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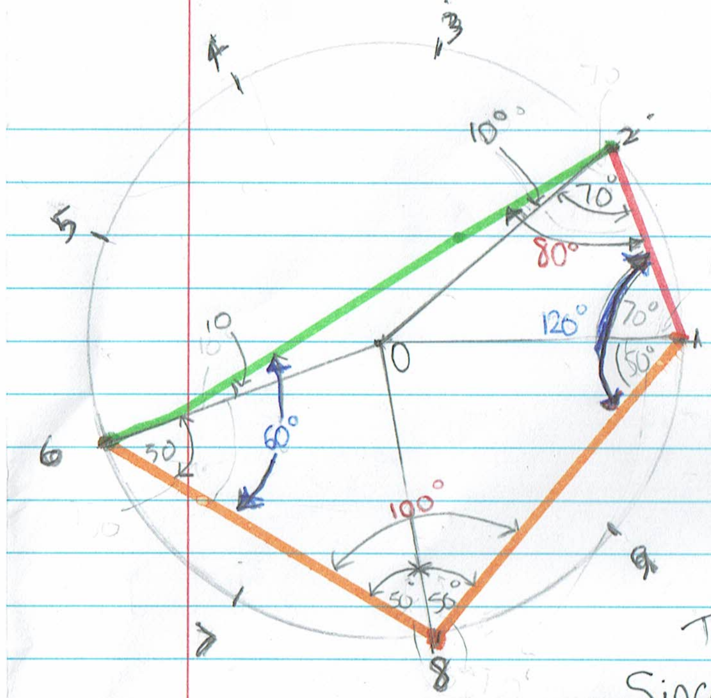
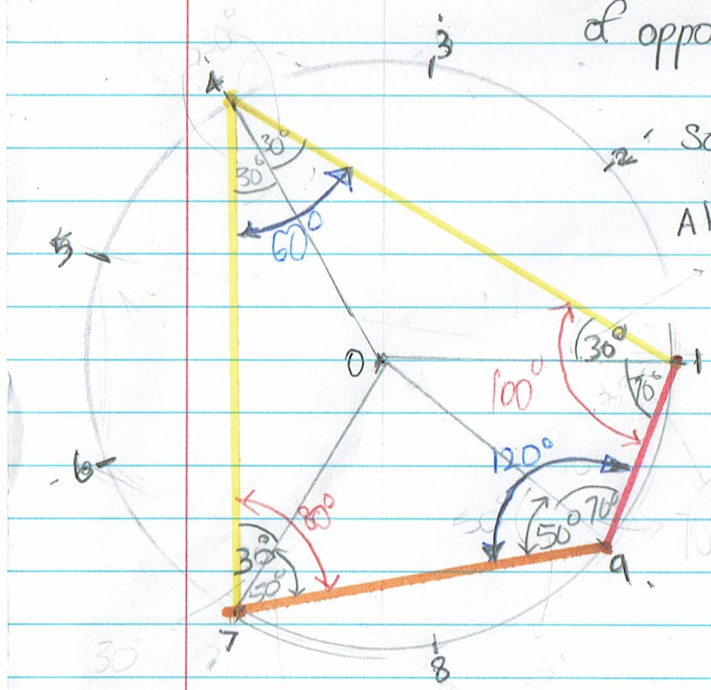


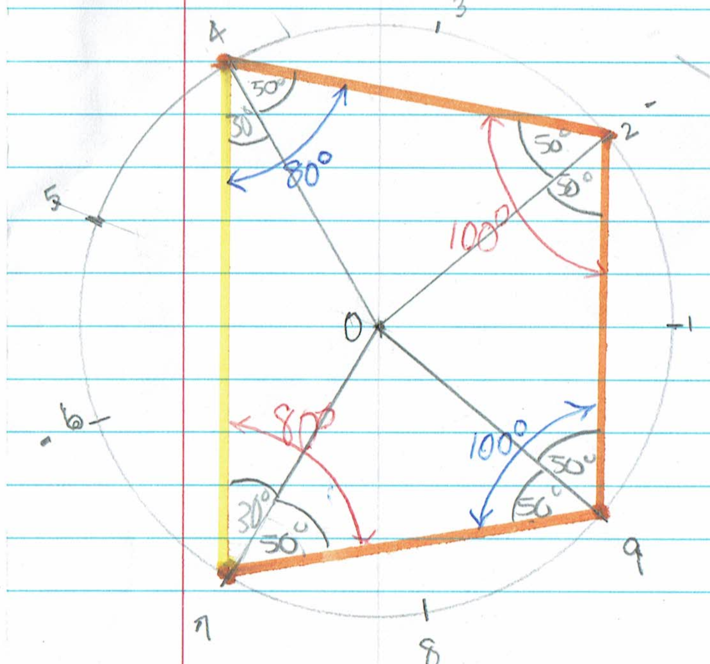
As shown here:

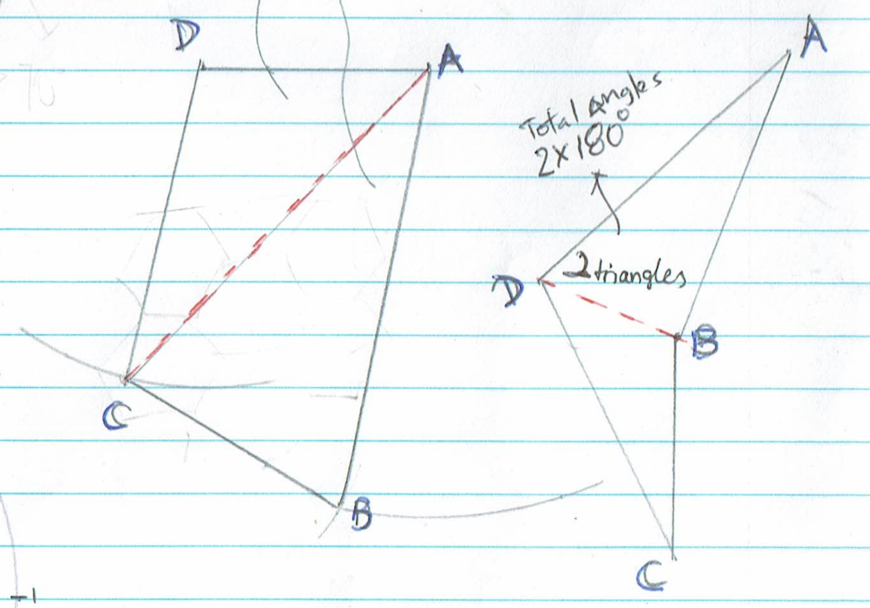
1) The opposite angles of a quadrilateral always add up to 180°.
2) The sum of all interior angles of the quadrilateral add up to 360°.

This can also be explained by the following:
Since there are four angles in total, and two pairs of opposite angles that add up to 180°,
$2 \times 180° = 360°$
∴ so all the angles add up to 360°.



Also, Total angles $\frac{2 \times 180°}{2\text{ triangles}}$ ∵ Each triangle will have a total internal angle sum of 180°





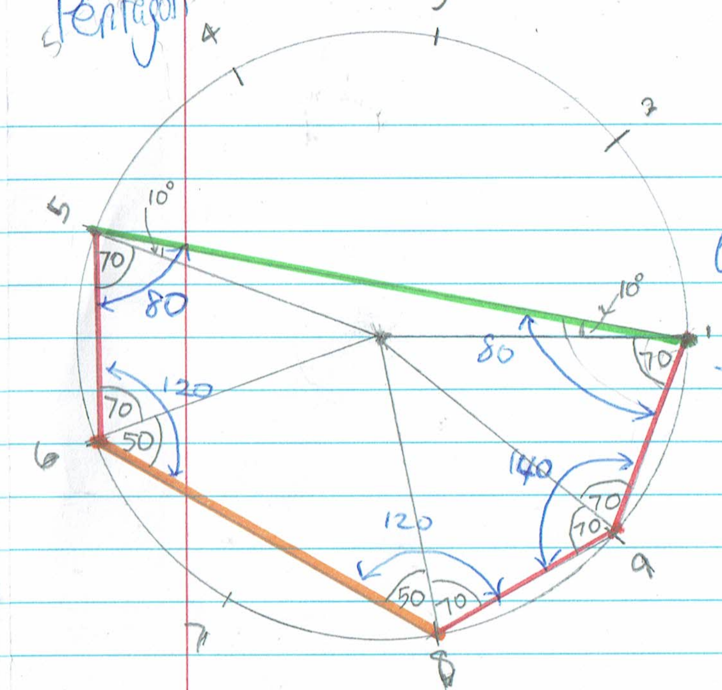
Quadrilateral

Generalisation

Sum of interior angles of a polygon

Polygon	Number of sides	Numbers of triangles formed	Sum of interior angles
Triangle	3	1	$1 \times 180° = (3-2) \times 180$
Quadrilateral	4	2	$2 \times 180° = (4-2) \times 180$
Pentagon	5	3	$3 \times 180° = (5-2) \times 180$
Hexagon	6	4	$4 \times 180° = (6-2) \times 180$
n-gon	n	n-2	$(n-2) \times 180°$

Pentagon

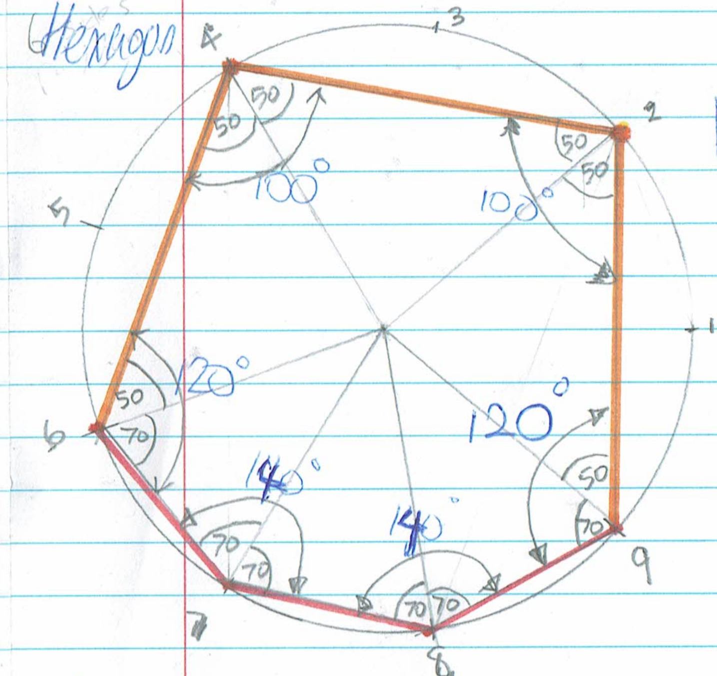


Odd number of interior angles.
100 "opposite" rules apply

Total Internal Angle

Pentagon = 540

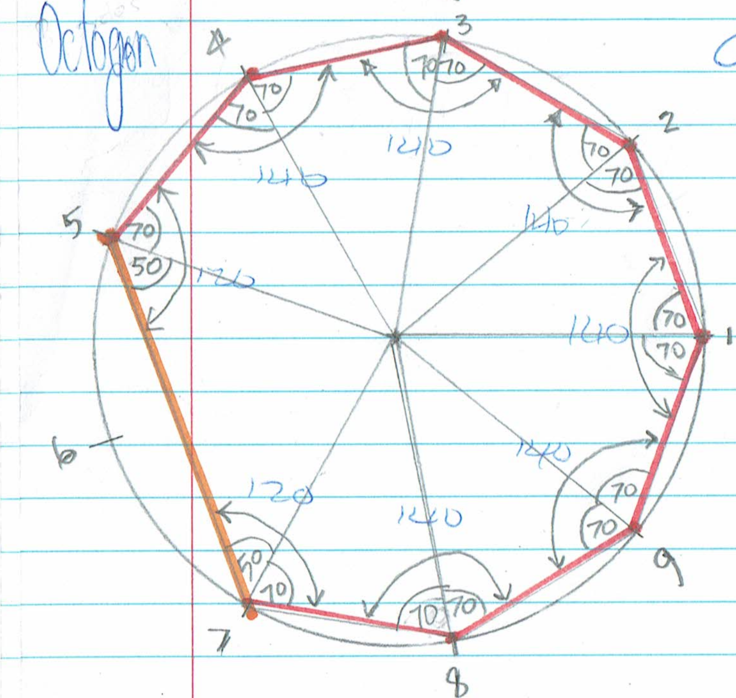
Hexagon



Hexagon = 720°

Sum of opposite pairs of interior angles are greater than 180°

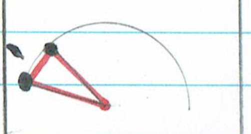
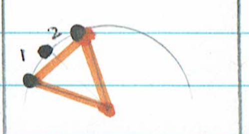
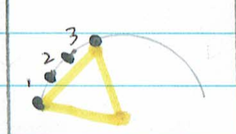
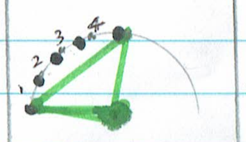
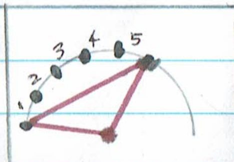
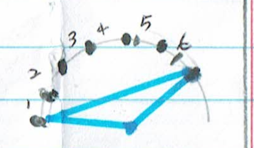
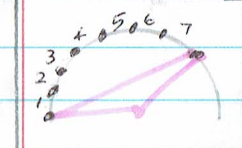
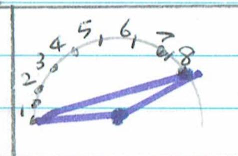
Octogon



Octogon = 1080°

Sum of opposite pairs of interior angles are greater than 180°

Types of Triangles

No. of Triangles formed on circle / Dots on Circle	 1 sector	 2 sectors	 3 sectors	 4 sectors	 5 sectors	 6 sectors	 7 sectors	 8 sectors
9	9	4	3	2				
10	10	5	3	2	nil			
12	12	6	4	3	2	nil		
15	15	7	5	3	3	2	2	
18	18	9	6	4	3	3	2	2
n	$n \div 1$	$\frac{n}{2} = Q+R$ Q	$\frac{n}{3} = Q_1+R_1$ Q_1	$\frac{n}{4} = Q_2+R_2$ Q_2	$\frac{n}{5} = Q_3+R_3$ Q_3	$\frac{n}{6} = Q_4+R_4$ Q_4	. . .	. . .
		Q = Quotient R = Remainder	Q' = Quotient Q' = Remainder	. . .	. . .	. . .		