Tiffany Lau — Senior Math Quiz : One Basket or Group Photo — 20-01-07

2 people

Let the two people be A and B (A is the shortest).
There is only 1 way of arranging them:

B	← 2nd row
A	← 1st row

4 people

The people are A, B, C and D (A is the shortest, D is the tallest).
Since the positions of A and D are fixed, there are only 2 ways:

C D	B D	← 2nd row
A B	A C	← 1st row

6 people

A, B, C, D, E and F, where the positions of A and F are fixed.

If A and B are fixed, there are 3 ways:

D E F	C E F	C D F
A B C	A B D	A B E

If A and C are fixed, there are 2 ways:

B E F	B D F
A C D	A C E

∴ Altogether there are $3+2 = 5$ ways.

8 people

A, B, C, D, E, F, G and H, where A and H have a fixed position.

If A, B and C are fixed, there are 4 ways:

E F G H	D F G H	D E G H	D E F H
A B C D	A B C E	A B C F	A B C G

If A, B and D are fixed, there are 3 ways:

C F G H	C E G H	C E F H
A B D E	A B D F	A B D G

If A, B and E are fixed, there are 2 ways:

C D G H	C D F H
A B E F	A B E G

Similarly, if A and C are fixed in the first 2 positions, there are $3+2 = 5$ ways:

B F G H	B E G H	B E F G	B D G H	B D F H
A C D E	A C D F	A C D G	A C E F	A C E G

$\therefore$ Altogether, there are $(4+3+2)+(3+2) = \underline{\underline{14 \text{ ways}}}$.

10 people

In the same way, if A, B and C are fixed in the first 3 positions, there will be $5+4+3+2 = \underline{12 \text{ ways}}$:

From
F G H I J
A B C D E
to
D E F H J
A B C G I

If A, B and D are fixed, there will be $4+3+2 = \underline{9 \text{ ways}}$:

From
C G H I J
A B D E F
to
C E F H J
A B D G I

If A, B and E are fixed, there will be $3+2 = \underline{5 \text{ ways}}$:

From
C D H I J
A B E F G
to
C D F H J
A B E G I

Then, if A, C and D are fixed in the first 3 positions, there will be $4+3+2 = \underline{9 \text{ ways}}$:

From
B G H I J
A C D E F
to
B E F H J
A C D G I

Finally, if A, C and E are fixed, there will be $3+2 = \underline{5 \text{ ways}}$:

From
B D H I J
A C E F G
to
B D F H J
A C E G I

$\therefore$ Altogether, there are $(5+4+3+2)+(4+3+2)+(3+2)+(4+3+2)+(3+2) = \underline{\underline{42 \text{ ways}}}$.

Tabulating all of these results,

n people	2	4	6	8	10
x ways	1	2	5	14	42

Notice that the formula for successive x values is $x_{a+1} = 3x_a - 1$,
but this formula breaks down when n reaches 10.

After trial and error, I have calculated the formula for x in terms of n, for $n=2$ to 8:

$$x = \frac{(3^{\frac{n}{2}-1})+1}{2} \quad \text{for } n=2, 4, 6 \text{ and } 8.$$