

This was a really fun problem to solve. I imagined the shape unfurl and rotate into a series of expanding rectangles in space. I visualised a series of images (played backwards and forwards) to understand how the angles and planes interact.

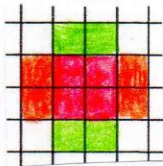
To be successful, the shape must maintain rotational symmetry, whilst it is expanding. The following is an explanation of how to draw the net on paper, the pictures say it all.

1)



I know there are 4 squares on the bottom, because the question says so.

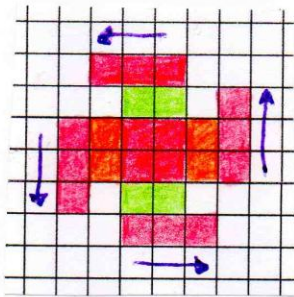
2)



On each side, there is a 2x1 strip of either orange or green. This represents the first vertical rise.

If you fold the sides up, they will meet at a 90° angle relative to each other and the base (a 3-d join; imagine looking into an empty box).

3)

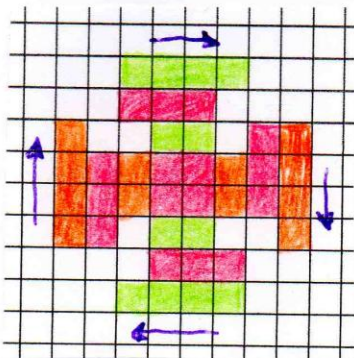


Now add a 3x1 strip of pink to the orange and green strips to represent the horizontal part of the first step (called a tread, the bit you stand on).

Because you are adding a 3x1 strip to a 2x1 strip, two squares match, with one extra. Arrange the extra square anticlockwise so that the shape has rotational symmetry (picture makes this clear)

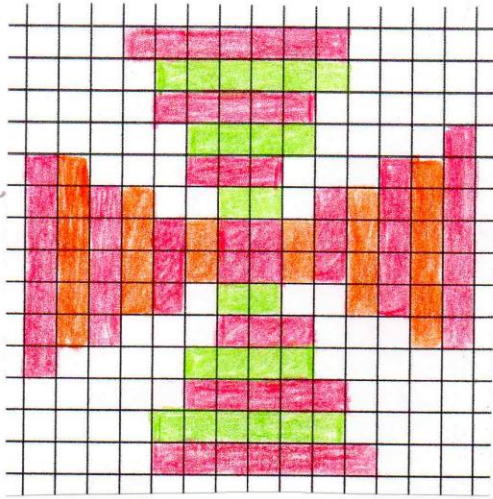
If you fold along the lines where the colours meet and open this up, you will now have a pink base, green/orange vertical sides and a pink horizontal first step.

4)

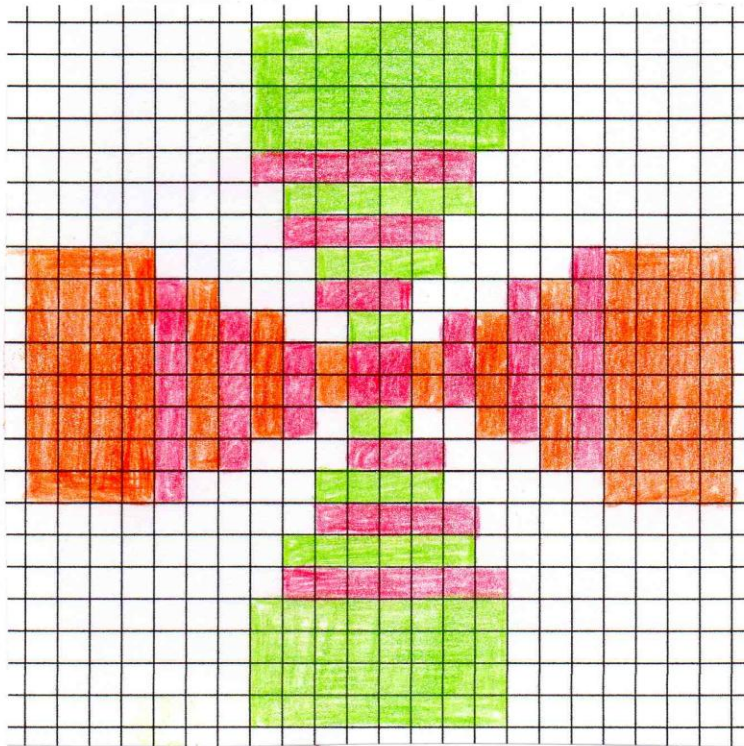


Then add a 4x1 strip but this time, arrange the extra square clockwise, so that the shape continues to have rotational symmetry.

If you fold along the lines where the different colours meet (coloured sides together this time), and open this out to 90°, you get the second vertical rise.



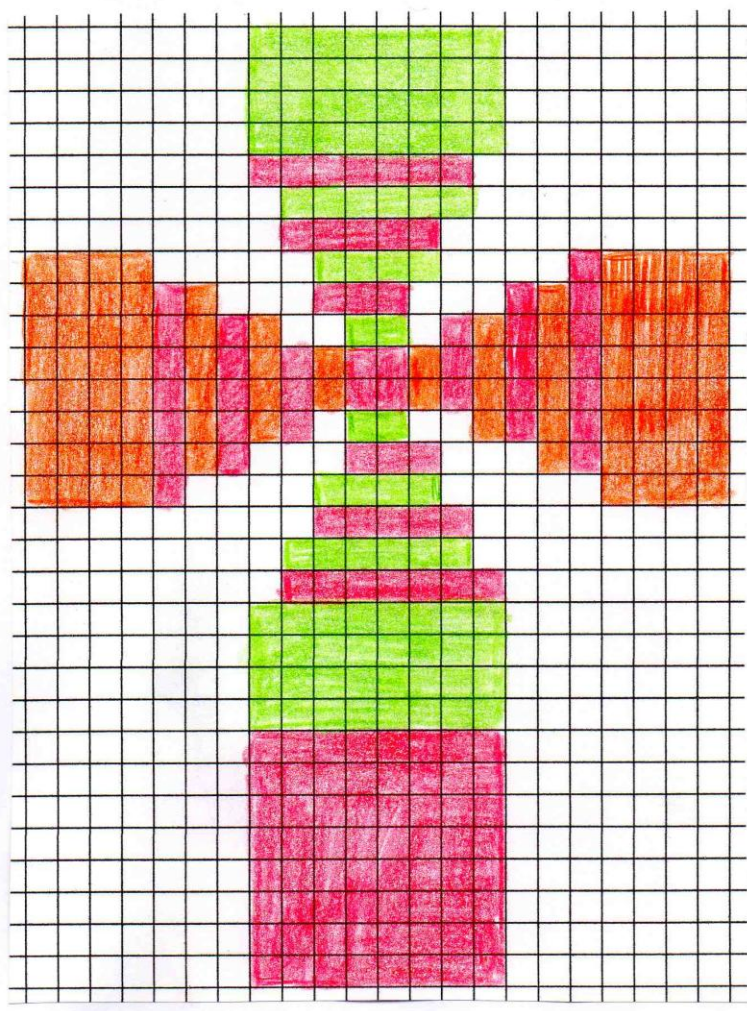
I then repeated steps 3) and 4), adding an extra square to each strip (for the expansion), and continued to add these in alternate anticlockwise and clockwise directions until the third pink horizontal step.



Finally, I just added the external 4x8 orange and green walls. The extra squares are arranged in a clockwise direction.

This is the net of the visible faces in 2d, as seen from above, but it does not include the external base of the auditorium (the face that sits on the ground) which is not visible here or in the picture online.

If you wanted to wrap the model as a gift, then you would have to include the external base. This no longer expands, but is simply an 8x8 pink square that I attached to one of the existing green walls (but it could have been any one)



Only now does the shape finally lose rotational symmetry.

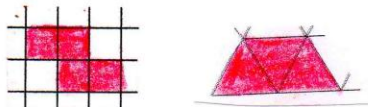
I started with 4(base), added 8(first rise), then 12(first step), etc... The extra 4 for the first step are needed, one for each corner. This then continued in multiples of 4 (because there are 4 sides), leading to a growth rule of $4n$. It's only because the model stopped expanding at row 7 that the pattern changed to accommodate the external walls and base.

The total surface area is 304 units^2 , the capacity of the void is 56 units^3 and the volume of the auditorium structure itself is 200 units^3 (assuming it's a solid construction).

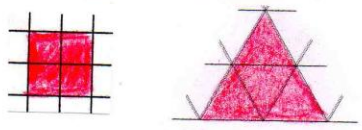
Term (row) number	1	2	3	4	5	6	7	8	9
Strips	4(1x1)	4(1x2)	4(1x3)	4(1x4)	4(1x5)	4(1x6)	4(1x7)	4(4x8)	1(8x8)
Total units	4	8	12	16	20	24	28	128	64

I used the same principle of adding to the corner to each strip and arranged these alternately anticlockwise and clockwise whilst the shape was growing to create other models. I thought of many complicated bases, but when I experimented and tried to draw the nets (and 'build' these), many failed.

I realised that if I could successfully draw a net to the first horizontal step, I could probably go on, set a pattern and continue to build. Also, I found that in the examples I tried, the base itself needed to have a rotational symmetry greater than 1. So for example, these bases failed:

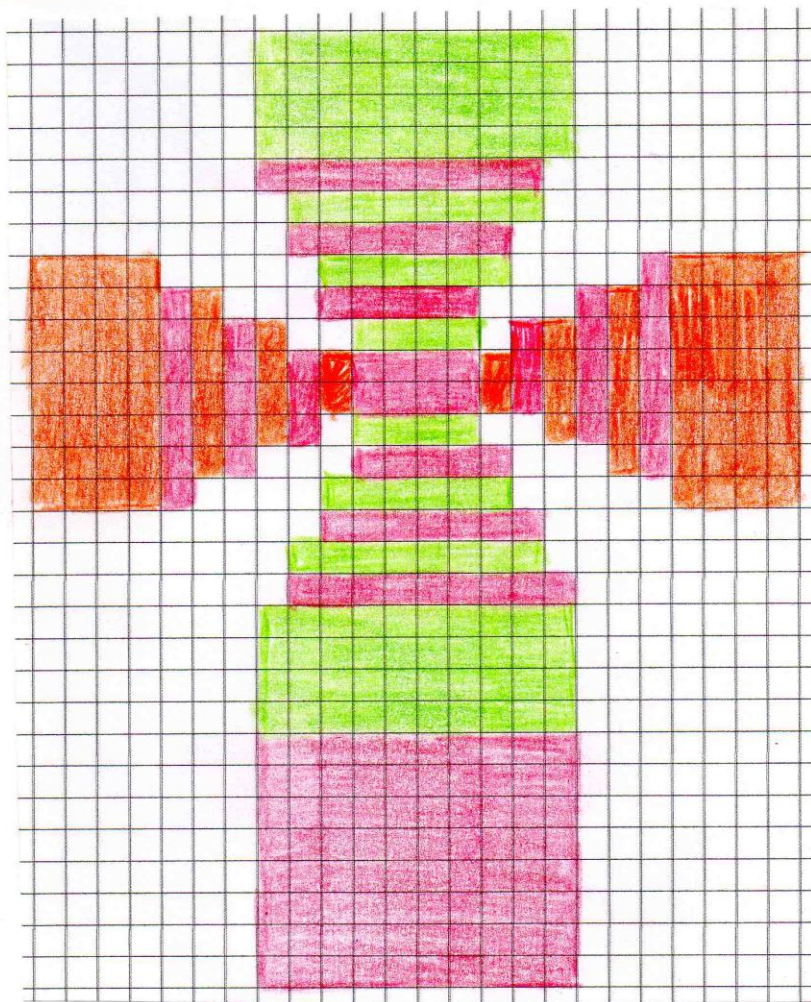


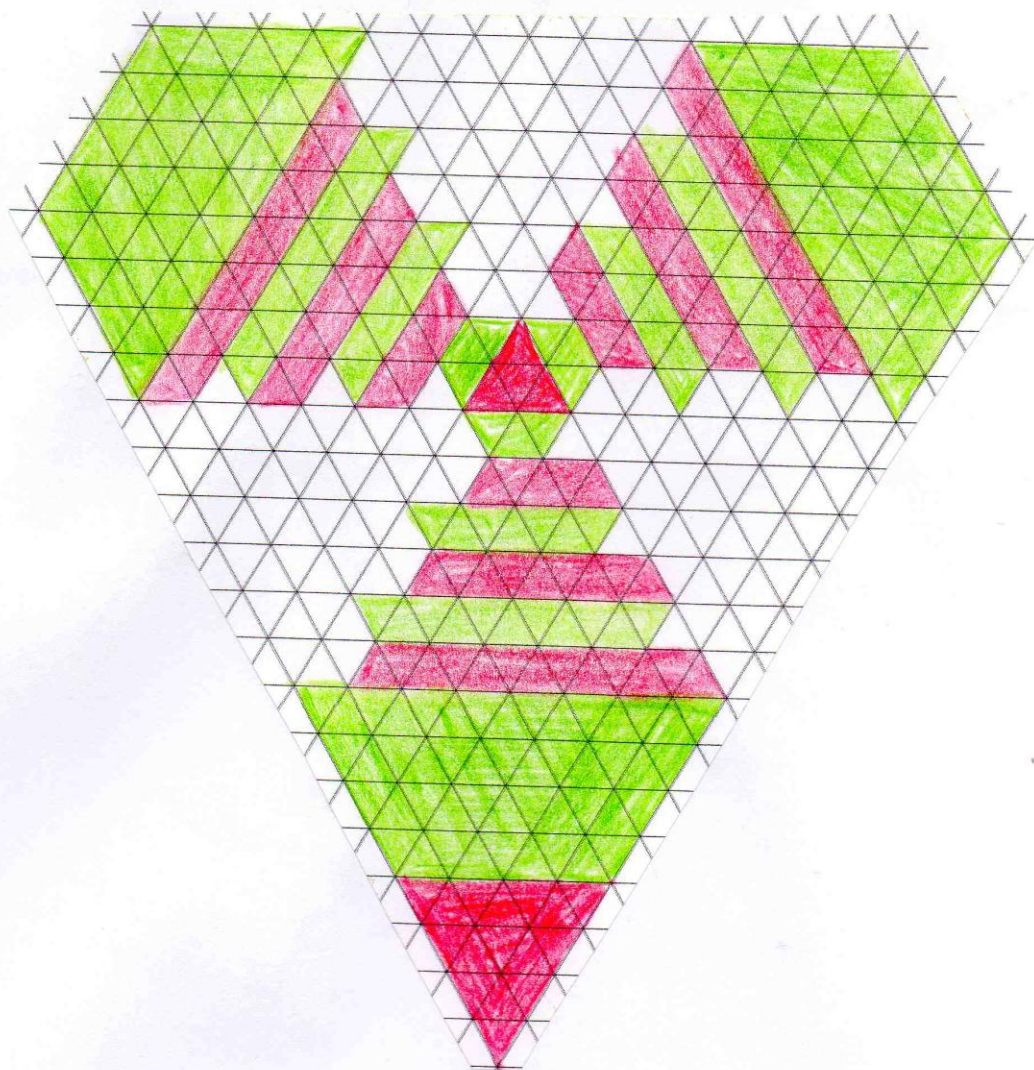
But these bases seemed to work:



I think it's all about angles, rotational symmetry and how shapes tessellate: basically you're aiming for sheets of tessellated shapes that can join at 90° angles (to safely create 'steps'). I used regular polygons, but it would be interesting to experiment with other shapes.

These are some of mine that worked. The first one is obvious: it's just a variation on the question. The second is a bit more complicated.





I predict that this shape should also work:

Internal base: regular hexagon

First rise: 6 squares

First step (tread): 6 x (square plus an equilateral triangle)

Second rise: 12 squares

Second step: 6 x (square plus 3 equilateral triangles arranged as a trapezium)

Third rise: 18 squares

Third step: 6 x (two squares arranged as a rectangle, plus 3 equilateral triangles arranged as a trapezium)

Side walls: 4 x 6 x 4 squares

External base: 24 squares, 42 triangles, 1 hexagon

But it's too complicated to draw on paper. You could always swap the hexagon for 6 equilateral triangles, so that the shape is just made out of two polygons, if you prefer.

This was really fascinating and I would love to learn more about the maths behind these structures. If you could programme an animation to manipulate this, it would be brilliant fun to play with!