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Pythagorean Triples Age 9

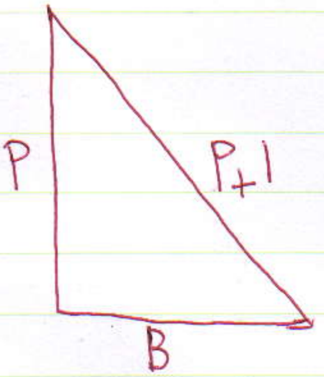
In Charlie's triples

$(P+1)^2 = P^2 + B^2$

or $P^2+1+2P = P^2+B^2$

or $2P+1 = B^2$

or $B = \sqrt{2P+1}$



$P, B \in \mathbb{R}, P, B \geq 0$

$\therefore$ Perpendicular $= P$, Hypotenuse $= P+1$, Base $= \sqrt{2P+1}$.
The above proves this.

In Alison's triples

$(P+2)^2 = P^2 + B^2$

or $P^2+4+4P = P^2+B^2$

or $B^2 = 4+4p$

or $B = \cancel{2+2p}\ 2\sqrt{1+p}$

$\therefore$ Perpendicular $= P$, Hypotenuse $= P+2$, Base : $2+2p = 2(1+p)$
The above proves this.

When $(P+n)^2 = P^2 + B^2$

$B^2 = n^2 + 2Pn$

or $B = \sqrt{n(n+2P)}$

$\therefore$ Perpendicular $= P$ Hypotenuse $= P+n$ Base $= \sqrt{n(n+2P)}$

Here $P \not< 0$

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If $B \in \mathbb{Z}^+$

Then in Charlie's triples

$2P+1 = x^2$, $x \in \mathbb{Z}^+ \therefore P \in \mathbb{Z}^+$ when x^2-1 is even and $P \notin \mathbb{Z}^+$ if x^2-1 is

In Alison's triples

$P \in \mathbb{Z}^+$ when $(P^2-4) \% 4 = 0$

And in general

$n^2 + 2Pn = x^2$, $x, n \in \mathbb{Z}^+$

$\therefore P \in \mathbb{Z}^+$ if $(x^2-n^2) \% 2n = 0$