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Which Is Cheaper?

Which car park should I use?

The total cost for parking in car parks A and B can both be modelled by the equations below:
$C_{car\ park\ A} = 80 + 50(x - 1)$
$C_{car\ park\ B} = 150 + 30(x - 1)$

Where C is the total car parking cost in pence and x stands for the total number of hours parked.
Representing this information in a simple table for up to 10 hours parked produces this:

Hours Parked	Cost for Car Park A (pence)	Cost for Car Park B (pence)
1	80	150
2	130	180
3	180	210
4	230	240
5	280	270
6	330	300
7	380	330
8	430	360
9	480	390
10	530	420

Letting x represent the number of hours parked for, the above table clearly shows that if $x \leq 4$ Car Park A should be selected and if $x \geq 5$ Car Park B should be selected.
Alternatively, given that both equations correspond to linear graphs in the form $y = mx + c$, the x coordinate of the intersection point can be determined by setting the two equations equal to each other:
$80 + 50(x - 1) = 150 + 30(x - 1)$
$20x = 90$
$x = 4.5$
We make the assumption that x takes an integer value so the inequalities $x \leq 4$ for Car Park A and $x \geq 5$ for Car Park B are established.

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What advice would you give me if I was trying to decide whether to use one of the car parks, the Park and Ride, or the train?

The total cost for using the Park and Ride service can be modelled by the equation below: $C_{park\ and\ ride} = 40x + 60$ where C is total cost in pence and x is number of hours parked in the Park and Ride.

The total cost for using the train is a constant price of 350p, therefore $C_{train} = 350$
Tabulating the total cost for both the Park and ride services as well as the train produces:

Hours Parked	Park and Ride (pence)	Train (pence)
1	100	350
2	140	350
3	180	350
4	220	350
5	260	350
6	300	350
7	340	350
8	380	350
9	420	350
10	460	350

We could consider all four total cost equations graphically however given the large gradients, this would render the graph hard to interpret as the lines would merge as we continue to zoom out of the graphing tool in order to determine intersection points of the four lines.

Comparing the two tables, we can establish the following conclusions:

1. If you want to park for more than 7 hours, $x > 7$, you should park for free at the railway station and the train to Mathstown should be taken.
2. If you want to park for up to three hours, $x \leq 3$, you should park your car at Car Park A. (Note that if you want to park for exactly three hours, $x = 3$, the Park and Ride service could also be used alongside Car Park A given that both have the same cost for three hours worth of parking.
3. If you want to park your car for 4 to 6 hours, $4 \leq x \leq 6$, the Park and Ride service should be used.

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4. If you want to park for exactly 7 hours, $x = 7$, Car Park B should be where you park your car. (Note that if you want to park for exactly 6 hours Car park b would be the joint cheapest option along with the Park and Ride service.

Here are some challenges to try.
Can you work out what the charging structure might be in each case?

Let x represent the number of hours parked

- **Two car parks A and B, such that car park A is cheaper if you park for less than 5 hours and car park B is cheaper if you park for more than 5 hours.**

  Graphically, Car Parks A and B would be linear graphs. The charging structure for Car Park A would be a multiple of $x - 5$ and the charging structure for Car Park B would be a multiple of $5 - x$ where x represents the number of hours parked.

- **Three car parks C, D and E, such that car park C is cheapest if you park for less than 2 hours, car park D is cheapest if you park for between 2 and 6 hours, and car park E is cheapest if you park for more than 6 hours.**

  Graphically, Car Park C and E would be linear whereas Car Park D would be a positive quadratic with roots of $x = 2$ and $x = 6$ with a minimum or cheapest car parking cost occurring at values of x where $2 \le x \le 6$. The charging structure of car park C would be a multiple of $x - 2$, Car Park D would have a charging structure in the form of $(x - 2)(x - 6)$ and Car Park E would have a charging structure in the form of $6 - x$ and any multiples of this.

- **Three car parks F, G and H, such that car park F is cheapest if you park for less than 3 hours, car park G is the cheapest if you park for more than 3 hours, and car park H is never cheapest.**

  The charging structure of car park F would be a multiple of $x - 3$, Car Park G would have a charging structure in the form of $3 - x$ and multiples of this whilst Car Park H would have a charging structure in the form of x and any multiples of this.

- **Three car parks I, J and K, such that car park I is always cheaper than car parks J and K, regardless of how long you park.**

  Graphically, all these lines would be parallel and therefore the cheapest one is the one with the lowest y-intercept. Thus, the charging structures of these three car parks could come in the form of x, $x+1$ and $x+2$ and any corresponding multiples of these three expressions.