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Which Is Bigger?

Which is bigger, n+10, or 2n+3? How did you decide?

To determine which expression is bigger at what particular values of n, I decided to form two inequalities - one where we take n+10 as the bigger expression and the other takes 2n+3 as the bigger one:

$n + 10 > 2n + 3$ (1)

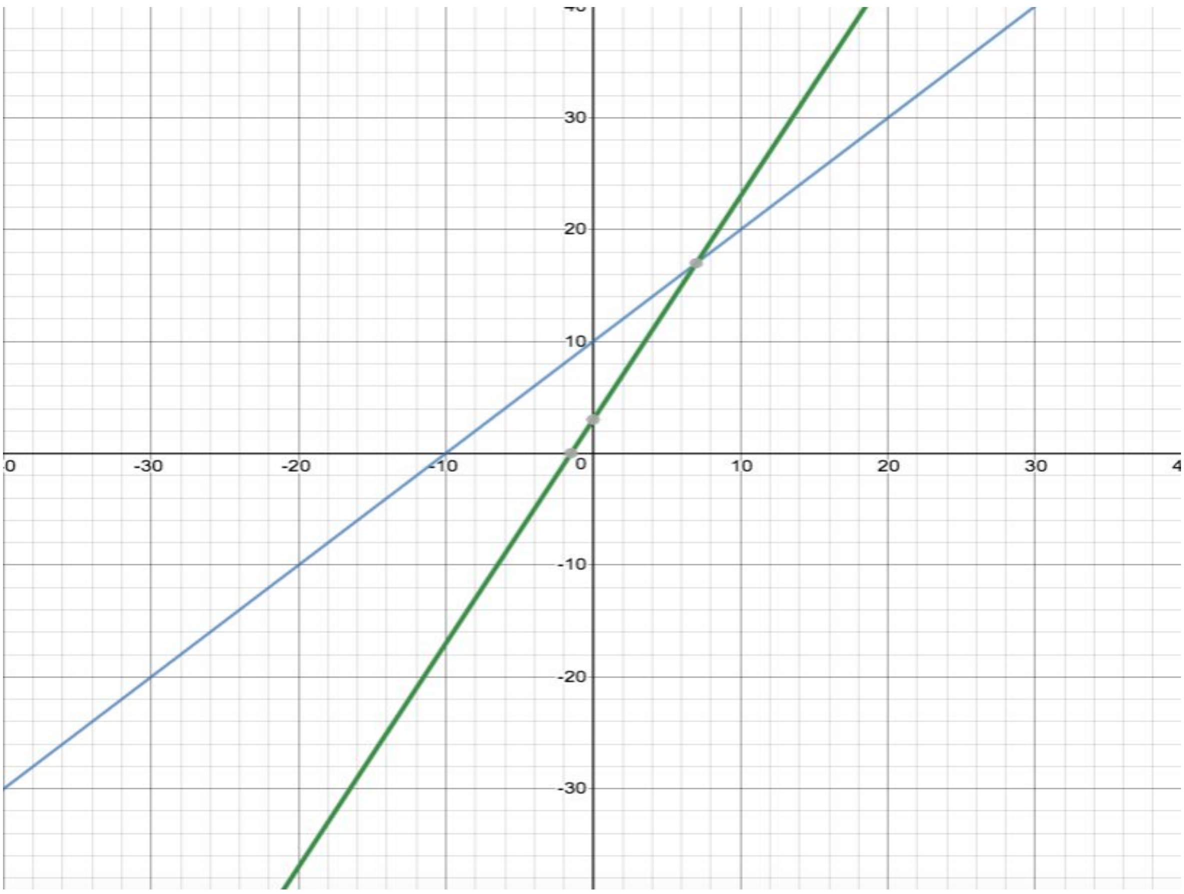
$n < 7$

$2n + 3 > n + 10$ (2)

$n > 7$

From the above inequalities , (1) and (2), we can conclude that when $n < 7$, $n + 10$ is the greater expression and when $n > 7$, $2n + 3$ is the bigger expression. Moreover, not that when n takes a value of 7, the two expressions are equal to each other.

Alternatively, we could plot the lines $n + 10$ and $2n + 3$ if we consider them as straight lines which take the form $y = mx + c$ (the green line represents $2n + 3$ and the blue line represents $n + 10$)



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The graph above clearly shows that when $n < 7$, the expression $n + 10$ has a greater corresponding y value and is therefore bigger and when $n > 7$, $2n + 3$ has the greater corresponding y value and is thus the bigger of the two expressions.

Can you explain why we have come to different conclusions?

You have both arrived at different conclusions as the two of you have tested values for n either side of 7, where the equality or in the intersection of the two expressions occurs.

For the following pairs of expressions, can you work out when each expression is bigger?

a) **2n+7 and 4n+11**
b) **2(3n+4) and 3(2n+4)**
c) **2(3n+3) and 3(2n+2)**

Implementing a similar structure to the one used above, we can determine when each of the three pairs of expressions is bigger at particular values of n.

a) $2n + 7 > 4n + 11$

$-2n > 4$

$n < -2$

$4n + 11 > 2n + 7$

$2n > -4$

$n > -2$

Using the two inequalities above we can establish that for values where $n < -2$, the expression $2n + 7$ is greater and for values where $n > -2$, the expression $4n + 11$ is bigger.

b) 2(3n+4) and 3(2n+4)

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$2(3n+4) > 3(2n+4)$

$6n + 8 > 6n + 12$

If we subtract $6n$ from both sides of the inequality we are left with $8 > 12$ which is not true. Closer look at the expansion of the two expressions, $6n + 8$ and $6n + 12$ respectively, reveals that for whatever value n takes, $3(2n+4)$ will always be the greater of the two expressions given that it has the same coefficient of n as $2(3n+4)$, that being 6, but a bigger constant term as $12 > 8$.

c) 2(3n+3) and 3(2n+2)

Expanding both these expressions results in $6n + 6$ and $6n + 6$. As a result, there is no bigger expression in this instance as $2(3n+3)$ and $3(2n+2)$ are always equal, whatever the value n is given.

Here are some challenges to try:

- **Find two expressions so that one is bigger whenever *n* < 5 and the other is bigger whenever *n* > 5.**

To find two expressions that satisfy the above constraints, the two expressions must be equal to each other when n takes a value of 5:

For example, $n + 7$ and $2n + 2$ would be two feasible expressions

$n + 7 > 2n + 2$

$n < 5$

And,

$2n + 2 > n + 7$

$n > 5$

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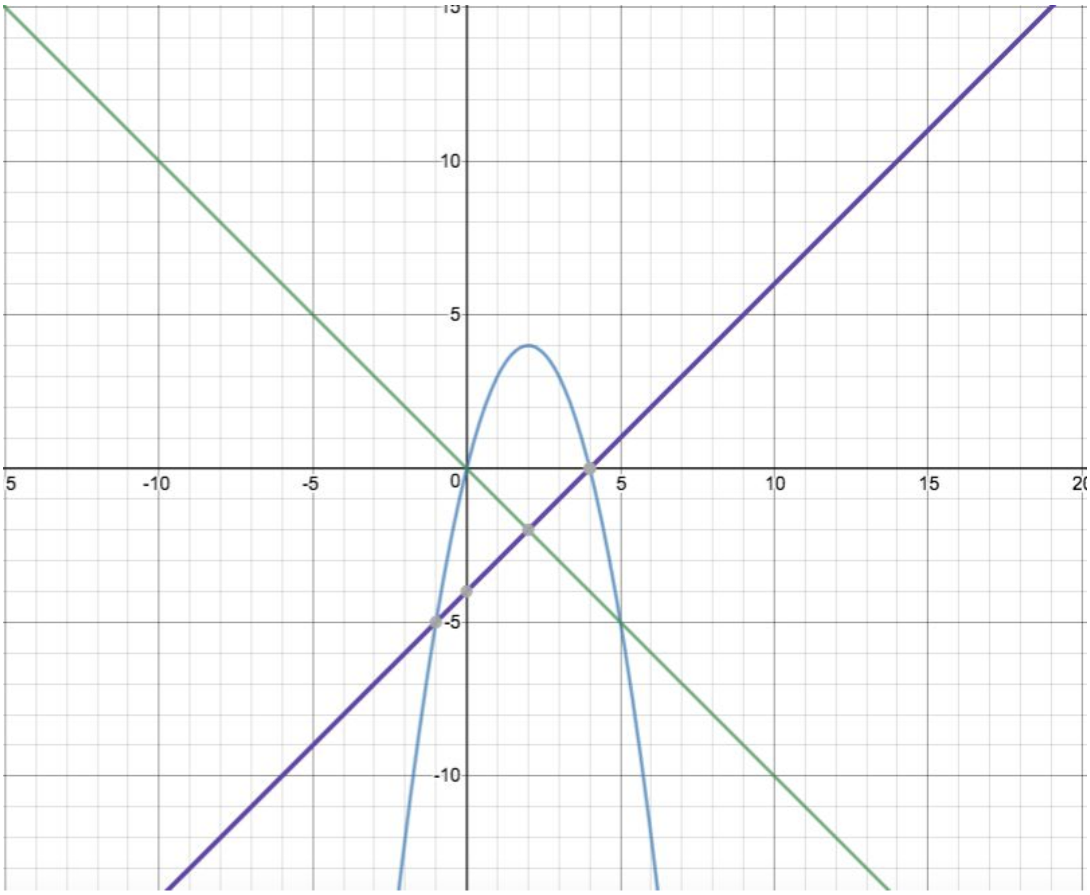
- **Find three expressions so that the first is biggest whenever $n < 0$, the second is biggest whenever n is between 0 and 4, and the third is biggest whenever $n > 4$.**

In this instance we have to consider quadratic expressions and their respective roots. In order to determine an expression which is the biggest when $0 < n < 4$, we should know that when the expression is set equal to 0 the two possible solutions, in this case for n, are 0 and 4 respectively. This establishes a quadratic with roots of $n = 0$ and $n = 4$. We must also determine the shape of the quadratic, whether it should be positive or negative, and in this case a maximum turning point is required and so a negative quadratic should be chosen. Therefore, following all the above constraints required for this quadratic expression we establish $-n(n-4)$ which can also be written as $4n - n^2$. The first expression which must be the biggest whenever $n < 0$ is $-n$ and the third expression which is the biggest whenever $n > 4$ is $n - 4$.

These three expressions can be shown graphically (the green line represents $-n$, the blue line represents $-n(n-4)$ and the purple line represents $n-4$)



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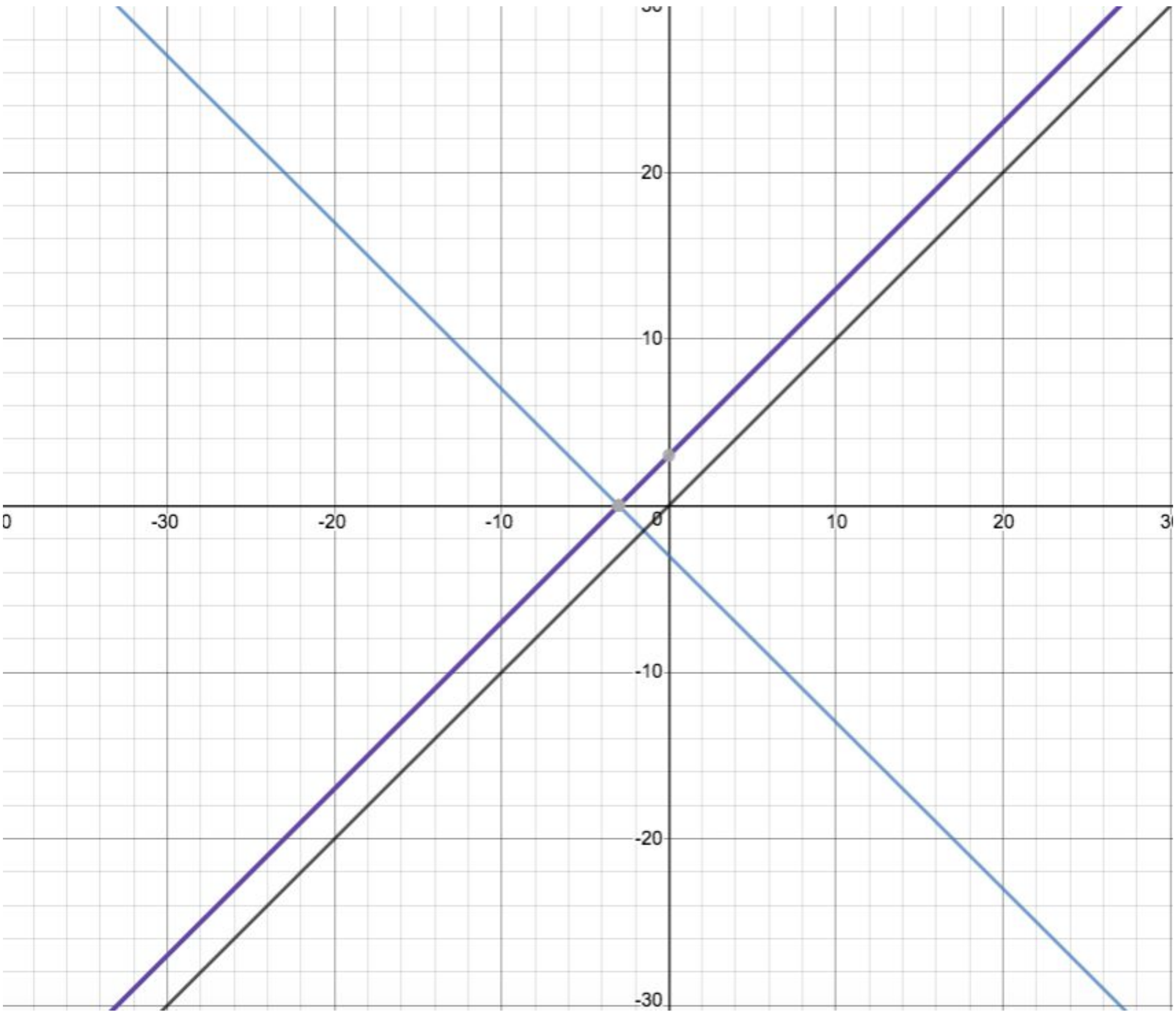
- **Find three expressions so that the first is biggest whenever *n*<3, the second is biggest when *n*>3, and the third is never the biggest.**

The three expressions in this scenario would be $-n-3$, $n+3$ and n. Graphically this produces the following (the blue line represents $-n-3$, the purple line represents $n+3$ and the black line represents n)



- **Find three expressions so that one of them is the biggest regardless of the value of *n*.**

There are many possible triplet of expressions which would satisfy the above constraints. For example, the expressions n, $n+1$ and $n+2$ would satisfy the constraints given that the expression $n+2$ is always the biggest whatever value n takes.