

Button up some more

3 buttons:

123 213 312
132 231 321

6 possibilities

4 buttons:

1234
1243
1324
1342
1423
1432

6 possibilities starting with 1.

Thus in total $4 \cdot 6 = 24$ possibilities

5 buttons:

$5 \cdot 24 = 120$ possibilities

N buttons:

$N!$ possibilities

$N=6:$ $6! = 720$

$N=7:$ $7! = 5040$

$N=8:$ $8! = 40320$

$N=9:$ $9! = 362880$

$N=10:$ $10! = 3628800$

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