Let's assume Rudolph's nose is the same size as a tennis ball.
Then his nose has volume $\frac{0.033^3 \times 4\pi}{3} \approx 1.51 \times 10^{-4} m^3 = 0.151 L \approx 150 ml$.
So in order to glow, we need 1.5×10^{23} molecules in his nose.

Since X halves every 10 minutes, X decreases by a factor of 1.0718 per minute.

Suppose we have Y bacterial cells.
As Y doubles every half an hour, Y increases by a factor of $\sqrt[30]{2} \approx 1.0234$ per minute.
X also increases by Y per minute as molecules are released.

Hence we find that:

$$\frac{dY}{dT} = 1.0234Y \quad (i)$$

$$\frac{dX}{dT} = \frac{X}{1.0718} + Y \quad (ii)$$

To solve (i), suppose $Y = Ae^{\lambda T}$.

$\Rightarrow \lambda Ae^{\lambda T} = 1.0234e^{\lambda T}$
$\Rightarrow \lambda = 1.0234$ (auxillary equation)
$\Rightarrow Y = Ae^{1.0234T}$ (general solution)
$\Rightarrow Y = e^{1.0234T}$ (particular solution as $Y=1$ at $T=1$)
$\Rightarrow \frac{dX}{dT} = \frac{X}{1.0718} + e^{1.0234T}$ (from (ii))

To solve this, first consider $\frac{dX}{dT} = \frac{X}{1.0718}$.
Trialling $X = Ae^{\lambda T}$,

$$\lambda Ae^{\lambda T} = \frac{Ae^{\lambda T}}{1.0718}$$

$\Rightarrow \lambda = 0.933$
$\Rightarrow X = Ae^{0.933T}$, $A=1$ as $X=1$ at $T=0$
$\Rightarrow X = e^{0.933T}$ (complimentary function)

Let's now consider $\frac{dX}{dT} = e^{1.0234T}$.
Integrating both sides:

$X = 0.977e^{1.0234T} + C$, $C=0$ as $X=1$ at $T=0$
$\Rightarrow X = 0.977e^{1.0234T}$ (particular solution)

Thus our general solution is:

$X = e^{0.933T} + 0.977e^{1.0234T}$
(which from (i) & (ii) is strictly increasing)

After 24 hours, $T = 6 \times 24 = 144$.
Therefore $X = 9.81 \times 10^{63}$ after 24 hours.
$9.81 \times 10^{63} > 1.5 \times 10^{23}$, so we have enough molecules.

Therefore Rudolph's nose will glow in time!