Multipacation Arithamagons

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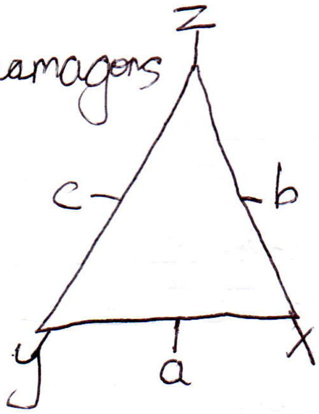
Q1.



The three edge numbers are a,b,c. Choose a pair of factors of a, let $a = xy$. Let us see if one of the factors is common with b, let x be common factor of a and b. Then in the vertex between a and b write x. In the vertex between a and c write y but if y is not a common factor of a, c then pair of factors x, y doesn't work. Then you have to repeat choice of factors of a. If y is common to a, c then write $\frac{c}{y} = z$. if $\frac{b}{x} \neq z$ then the pair of factors x, y does not work. Then you move on to the next pair of factors of a.

Q2. $\frac{\cancel{abc}\ xyz}{\cancel{xyz}\ abc} \quad \frac{xyz}{abc} \quad \frac{abc}{xyz}$

$= \frac{xy\ xz\ yz}{\cancel{abc}\ xyz}$

$= \frac{x^2y^2z^2}{xyz}$

$= xyz$

$\therefore$ Product of edge numbers is always the square of product of vertex numbers.

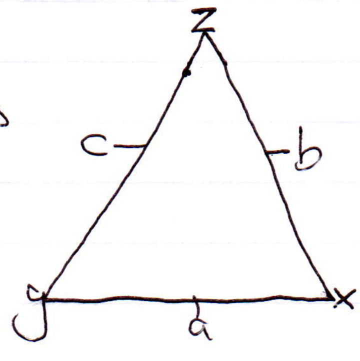
Q3. Case 1: Only edge number changing is a.
a is scaled by a factor of n, $n \in \mathbb{R}$, $n > 0$
~~The~~ The scale by which x and y have to be multiplied has to be the same so that z can compensate for the scaling.
∴ The square root of x and y is $\sqrt{n}$. This is why n should be possitive
∴ The scaling of z should be $\frac{1}{\sqrt{n}}$.

Case 2: Only edge numbers changing are a and b.
a and b are scaled by a factor of n each, $n \in \mathbb{R}$, $n > 0$.
y is scaled by a factor of g, $g > 0$.
x is scaled by a factor of k, $k > 0$.
Z is scaled by a factor of i, $i > 0$.

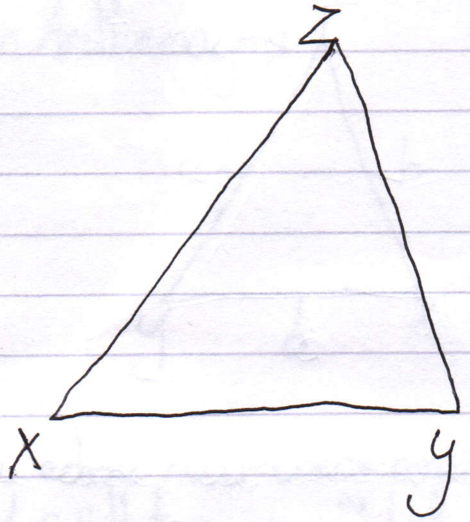
∴ $gk = n$ — ①
$ki = n$ — ②
$gi = 1$ — ③
① ÷ ② ⟹
$\frac{g}{i} = 1$ — ④
④ × ③ ⟹
$g = 1$
∴ $k = n$
$i = 1$
∴ ~~Only~~ Only x is scaled by a factor of n.

Case 3: When a, b, c are all scaled by a factor of n
x, y, z all scaled by $\sqrt{n}$

Q4. The only arithmagon possible is
~~$~~ $a, b, c \in \mathbb{Z}$
$y = \frac{1}{n}$, $n > 1$, $x = mn$, $m \in \mathbb{Z}$
$z = ln$, $l \in \mathbb{Z}$



Q5.



$\cancel{x \in \mathbb{R}}$ $\quad x = x^{\frac{1}{2}} \quad y = l x^{\frac{1}{2}} \quad z = m x^{\frac{1}{2}}$

$\cancel{y \in \mathbb{R}}$ $\quad x \in \mathbb{R} \quad l \in \mathbb{R} \quad m \in \mathbb{R}$

x can only be raised by half. Here's why:

$l x^{1-\frac{1}{n}} \; m x^{1-\frac{1}{n}} = lm$

$(x^{1-\frac{1}{n}})(x^{1-\frac{1}{n}}) = x^1$

$x^{2-\frac{2}{n}} = x^1$

$2 - \frac{2}{n} = 1$

or $2 = 1 + \frac{2}{n}$

or $1 = \frac{2}{n}$

$\therefore n = 2$

Q6. All three vertices can be irrational if the edge is rational