For part 1a) we have 1 square and 12 units along with many sticks.

Let the dimensions of rectangle be $(x + a)(x + b)$

$$(x + a)(x + b) = x^2 + (a+b)x + 12$$

$$\therefore ab = 12$$

a and b can be:

$$1 \times 12 = 12$$
$$2 \times 6 = 12$$
$$3 \times 4 = 12$$

$$(x + 1)(x + 12) = x^2 + 13x + 12$$
$$(x + 2)(x + 6) = x^2 + 8x + 12$$
$$(x + 3)(x + 4) = x^2 + 7x + 12$$

3 Solutions

For part 1b) we have 1 square and 100 units along with lots of sticks.

Similar to part 1a),

$$(x + a)(x + b) = x^2 + (a+b)x + 100$$

$$\therefore ab = 100$$

a and b can be:

$$1 \times 100 = 100$$
$$2 \times 50 = 100$$
$$4 \times 25 = 100$$
$$5 \times 20 = 100$$
$$10 \times 10 = 100$$

$$(x + 1)(x + 100) = x^2 + 101x + 100$$
$$(x + 2)(x + 50) = x^2 + 52x + 100$$
$$(x + 4)(x + 25) = x^2 + 29x + 100$$
$$(x + 5)(x + 20) = x^2 + 25x + 100$$
$$(x + 10)(x + 10) = x^2 + 20x + 100$$

5 Solutions

For part 2a) $(x + a)(x + b) = x^2 + 12x + ab$

In this example we have sticks as 12 and lots of unit

$a + b = 12$ $\quad$ $a \times b =$

$$0 + 12 = 12 \qquad 0 \times 12 = 0$$
$$1 + 11 = 12 \qquad 1 \times 11 = 11$$
$$2 + 10 = 12 \qquad 2 \times 10 = 20$$
$$3 + 9 = 12 \qquad 3 \times 9 = 27$$
$$4 + 8 = 12 \qquad 4 \times 8 = 32$$
$$5 + 7 = 12 \qquad 5 \times 7 = 35$$
$$6 + 6 = 12 \qquad 6 \times 6 = 36$$

$$(x + 0)(x + 12) = x^2 + 12x + 0$$
$$(x + 1)(x + 11) = x^2 + 12x + 11$$
$$(x + 2)(x + 10) = x^2 + 12x + 20$$
$$(x + 3)(x + 9) = x^2 + 12x + 27$$
$$(x + 4)(x + 8) = x^2 + 12x + 32$$
$$(x + 5)(x + 7) = x^2 + 12x + 35$$
$$(x + 6)(x + 6) = x^2 + 12x + 36$$

We could keep going for 7 + 5 and 8 + 4 and so on... But 7 x 5 = 5 x 7 and for rest as well which do not give a new rectangle.

<u>7 solutions</u>

For part 2b) $(x + a)(x + b) = x^2 + 100x + ab$

Similar to part 2a)

$a + b = 100$ $\quad$ $a \times b =$

$$0 + 100 = 100 \qquad 0 \times 100 = 0$$
$$1 + 99 = 100 \qquad 1 \times 99 = 99$$
$$2 + 98 = 100 \qquad 2 \times 98 = 196$$
$$3 + 97 = 100 \qquad 3 \times 97 = 291$$

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$$49 + 51 = 100 \qquad 49 \times 51 = 2499$$
$$50 + 50 = 100 \qquad 50 \times 50 = 2500$$

$$(x + 0)(x + 100) = x^2 + 100x + 0$$
$$(x + 1)(x + 99) = x^2 + 100x + 99$$
$$(x + 2)(x + 98) = x^2 + 100x + 196$$
$$(x + 3)(x + 97) = x^2 + 100x + 291$$

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$$(x + 49)(x + 51) = x^2 + 100x + 2499$$
$$(x + 50)(x + 50) = x^2 + 100x + 2500$$

<u>51 solutions</u>

We could keep going for 51 + 49 and 52 + 48 and so on... But 51 x 49 = 49 x 51 and for rest as well which do not give a new rectangle.

For a given number of sticks say ’n’ and 1 square in the same base along with lots of units, There can be a total of:

$\dfrac{n + 2}{2}$ Solutions for even ’n'

$\dfrac{n + 1}{2}$ Solutions for odd ’n'

For part 3), We convert the 1 square, p sticks and q units into an algebraic form of base 'x'

$$x^2 + px + q$$

Let the dimensions of the rectangle be $(x + a)(x + b) = x^2 + (a + b)x + ab$

Thus on comparison we get $\therefore p = a + b$

$$q = a\,b$$