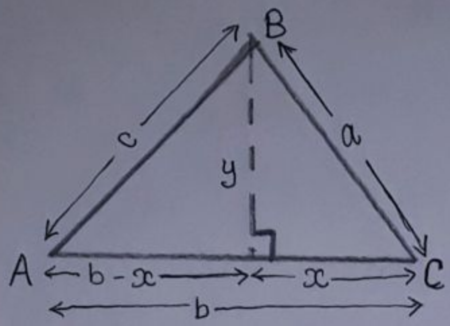
Q. Find a common equation for the side C from the given data?

Solution :



Student 1

By pythagoras theorem,

$$x^2 + y^2 = a^2 \Rightarrow y^2 = a^2 - x^2 \quad (1)$$

$$(b-x)^2 + y^2 = c^2 \Rightarrow y^2 = c^2 - (b-x)^2 \quad (2)$$

Equating (1) and (2), we get

$$a^2 - x^2 = c^2 - (b-x)^2 \Rightarrow a^2 + (b-x)^2 - x^2 = c^2 \quad (3)$$

From figure,

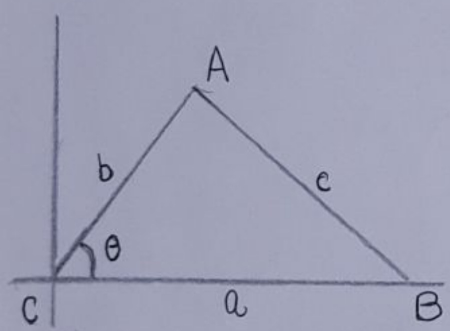
$$\cos C = \frac{x}{a} \Rightarrow x = a \cos C$$

Substing x in (3),

$$a^2 + (b - a\cos C)^2 + a^2 \cos^2 C = c^2$$

$$c = \sqrt{a^2 + b^2 - a^2\cos^2 C - 2ab\cos C + a^2\cos^2 C}$$

$$c = \sqrt{a^2 - 2ab\cos C + b^2}$$



Student 2

$A(b\cos\theta, b\sin\theta)$, $B(a, 0)$, $C(0,0)$

[Distance formula between two points (x_1, y_1) & (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$]

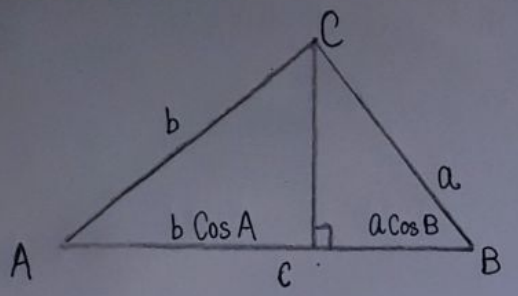
length of $c = \sqrt{(a - b\cos\theta)^2 + (0 - b\sin\theta)^2} = \sqrt{a^2 + b^2\cos^2\theta - 2ab\cos\theta + b^2\sin^2\theta}$

$[\sin^2\theta + \cos^2\theta = 1]$

$$c = \sqrt{a^2 - 2ab\cos\theta + b^2}, \quad \text{Since } \theta = \angle C$$

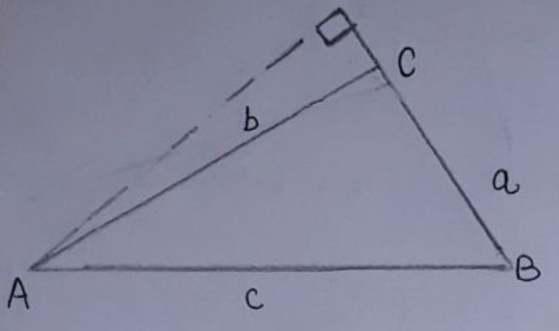
$$c = \sqrt{a^2 - 2ab\cos C + b^2}$$

Student 3



$$c = b\cos A + a\cos B$$

$$c^2 = bc\cos A + ac\cos B \quad (1)$$



$$a = c\cos B - b\cos(180^\circ - C)$$

$$a = c\cos B + b\cos(C)$$

$$a^2 = ac\cos B + ab\cos C \quad (2)$$

$$b^2 = ab\cos C + bc\cos A \quad (3)$$

$$a^2 - ab\cos C = ac\cos B \quad (4)$$

$$bc\cos A = b^2 - ab\cos C \quad (5)$$

Substituting (4) & (5) in (1)

$$c^2 = a^2 - ab\cos C + bc\cos A$$

$$c^2 = a^2 - ab\cos C + b^2 - ab\cos C$$

$$c = \sqrt{a^2 - 2ab\cos C + b^2}$$

$\therefore$ the common equation for side $c = \sqrt{a^2 - 2ab\cos C + b^2}$.

SOLVED BY

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