Exploring Cubic Functions

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Observations from the interactive graph: 1

First cubic graph has three roots. As we adjust the slider, one of the roots (x=0) stays the same. The other two roots symmetrically shift "a' units from the origin.

Also at a = 0, there are three repeated roots.

Hence the equation of the graph should $x(x - a)(x + a)$

Observations from the interactive graph: 2

In the second graph, using Vieta's results, we can say that since sum of roots is 0, the coefficient of x^2 will be 0.

For $p = 0$ and some arbitrary value of q, we see that the other two roots are shift q units. Similarly, when $q = 0$ and some arbitrary value of p, the first and the third roots are shifted p units.

Thus, the third root always shifts and is some combination of p and q

By experimenting and further observation, I think the function is of the form:
$f(x) = (x - p)(x - q)(x + p + q)$

After expansion, it yields this:

$f(x) = (x - p)(x - q)(x + p + q)$
$f(x) = x^3 + x^2(p + q - p - q) + x(pq - p^2 - pq - qp - q^2) + pq(p + q)$
$f(x) = x^3 + -x(p^2 + qp + q^2) + pq(p + q)$

Matching the functions

In the many functions given, If we assume the three roots to be α, β, γ then any general cubic would be of the form:

$$f(x) = x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \alpha\gamma) - \alpha\beta\gamma$$