Three Neighbours

Age 7 to 14
Challenge Level ★★

5, 6 and 7 are three consecutive numbers. They add up to 18.

14, 15 and 16 are also three consecutive numbers. They add up to 45.

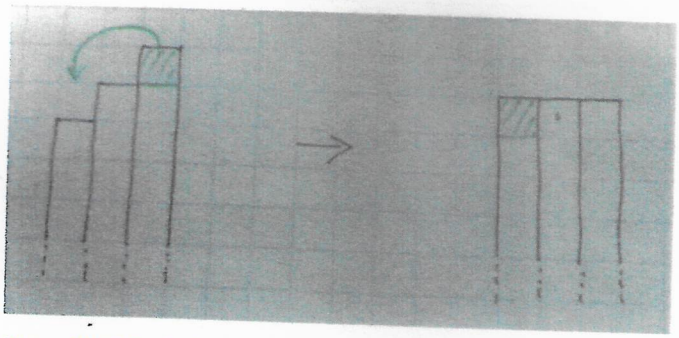
Take other sets of three consecutive numbers and find their total.

What do you notice?
Do the totals have anything in common?

How can you be sure that what you have noticed will always be true?

Mathematicians aren't usually satisfied with a few examples to convince themselves that something is always true.
Have you been able to provide an argument that would convince mathematicians?

Liz noticed that all the totals are a multiple of 3. She found it useful to draw a picture:



How did this help to convince Liz that three consecutive numbers always add up to a multiple of 3?

Charlie also noticed that all the totals were a multiple of 3. He thought about sets of numbers in a systematic way:

Hide Charlie's Method

Charlie started with the three consecutive numbers 1, 2, 3. They add up to 6.

He added 1 to each, which gave him the next three consecutive numbers 2, 3, 4. They add up to 9.

He added 1 to each again, and ended up with 3, 4, 5. They add up to 12.

How did this help to convince Charlie that three consecutive numbers always add up to a multiple of 3?

If you have met algebra before, then you might like to look at Claire's method:

Hide Claire's Method

Claire decided to call the first number n.

The next two numbers are then equal to $n+1$ and $n+2$.

Claire added the three numbers to get $n+n+1+n+2=3n+3$.

How did this help to convince Claire that three consecutive numbers always add up to a multiple of 3?

What happens when you add five consecutive numbers? Seven consecutive numbers? ...

Verbal feedback given

1+2+3 = 6

2 + 3 + 4 = 9

3 + 4 + 5 = 12

4 + 5 + 6 = 15

I notice that it goes up in the 3 x table. (e.g., 6, 9, 12, 15, 18, 21 ...)
They all are a ~~the~~ multiple of 3.

5 + 6 + 7 = 18

To convince the mathematicians, on each equation you ~~add~~ add 3 to the smallest number and keep the others. E.g. = 1 + 2 + 3, then 2 + 3 + 4

1 + 3 = 4

then it will carry on in the 3 times table.

Liz (in her diagram) ~~Subtracted the~~ She subtracked one and it becomes another multiple of 3.

To help convince Charlie, she in total added 3 to the answer, e.g $6+3=9$, $1+2+3=6$, $6+3=9$, $2+3+4=9$, then add 3 .....

To help convince Claire, she did '$n+1+n+2+n=3n+3$.' This is the same as Charlie's, because you are adding 3 to the end result.

$1+2+3+4+5=15$

$2+3+4+5+6=20$

$3+4+5+6+7=25$

When you add 5 consecutive numbers, the total is a multiple of 5, since each number in the sequence goes up in 5. This happens with other equations.