

Three Neighbours

Age 7 to 14
Challenge Level ★★

5, 6 and 7 are three consecutive numbers. They add up to 18.

14, 15 and 16 are also three consecutive numbers. They add up to 45.

Take other sets of three consecutive numbers and find their total.

What do you notice?

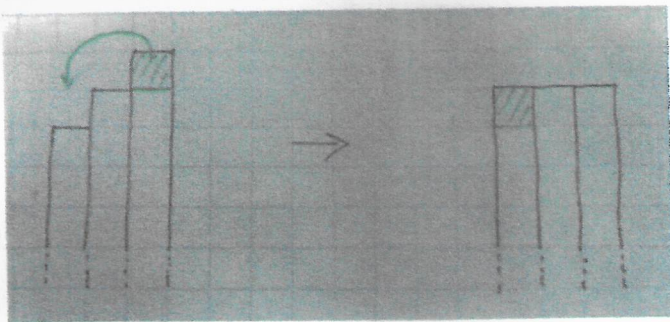
Do the totals have anything in common?

How can you be sure that what you have noticed will always be true?

Mathematicians aren't usually satisfied with a few examples to convince themselves that something is always true.

Have you been able to provide an argument that would convince mathematicians?

Liz noticed that all the totals are a multiple of 3. She found it useful to draw a picture:



How did this help to convince Liz that three consecutive numbers always add up to a multiple of 3?

Charlie also noticed that all the totals were a multiple of 3. He thought about sets of numbers in a systematic way:

Hide Charlie's Method

Charlie started with the three consecutive numbers 1, 2, 3. They add up to 6.

He added 1 to each, which gave him the next three consecutive numbers 2, 3, 4. They add up to 9.

He added 1 to each again, and ended up with 3, 4, 5. They add up to 12.

How did this help to convince Charlie that three consecutive numbers always add up to a multiple of 3?

If you have met algebra before, then you might like to look at Claire's method:

Hide Claire's Method

Claire decided to call the first number n .

The next two numbers are then equal to $n + 1$ and $n + 2$.

Claire added the three numbers to get $n + n + 1 + n + 2 = 3n + 3$.

How did this help to convince Claire that three consecutive numbers always add up to a multiple of 3?

What happens when you add five consecutive numbers? Seven consecutive numbers? ...

$$\begin{array}{l}
 1 + 2 + 3 = 6 \\
 2 + 3 + 4 = 9 \\
 3 + 4 + 5 = 12 \\
 4 + 5 + 6 = 15
 \end{array}$$

All in 3 times table

I notice that all the totals are in the

3 times table. This is always what they have in common.

This will always be the same because the difference between each number going down like this * is 1. $1 + 1 + 1 = 3$ so the difference between each total will be 3.

Yes I believe I have provided an argument strong enough to convince mathematicians.

Liz

If you add a the same number 3 times you get a multiple of 3 so she took 1 of the largest number and added it to the



Smallest number to make them all equal. ✓

Charlie

If the first 3 numbers which are 1, 2 and 3 add up to 6 and then you add 1 to all the numbers it is the same as $6 + 3$ which is ~~more~~ 9. This pattern will continue and every total will be the next multiple of 3. E.g. $6 + 3 = 9 + 3 = 12 + 3 = 15$ etc... ✓

Clare

If, for example $n=2$ $2+1=3$ $2+2=4$ and $3 \times 2 + 3 = 9$. This will always result in a multiple of 3 because any number multiplied by 3 will be a multiple of 3 and if you add 3 again it will still be a multiple of 3. Clare's method is similar to Charlie's because you are adding three ~~at the end~~ to the total you get.

$$1+2+3+4+5=15$$

$$2+3+4+5+6=20$$

$$3+4+5+6+7=25$$

$$1+2+3+4+5+6+7=28$$

$$2+3+4+5+6+7+8=35$$

$$3+4+5+6+7+8+9=42$$

This is the same as 3 consecutive numbers but always with a difference of the number of consecutive numbers you're adding.