Age 9

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Seven Squares

Phoebe - Phoebe drew 1 'down' at the beggining and 7 'inverted C's' to draw 7 squares. As each 'inverted C' consists of 3 matchsticks and 1 down uses 1 matchstick, altogether Phoebe uses $1+(7\times3)=22$ matchsticks.
∴ When drawing n squares Phoebe will draw 1 'down' and n 'inverted C's' ⇒ she will use $1+3n$ matchsticks.
∴ For drawing 25 squares she will use 76 matchsticks and for 100 squares 301 matchsticks.

Alice - Alice drew 14 'alongs' and 8 'downs' ⇒ she used $14+8=22$ matchsticks.
∴ When drawing n squares Alice will have to draw 2n 'alongs' and n+1 'downs' ⇒ she will use $n+1+2n=1+3n$ matchsticks.
∴ For drawing 25 squares, she will use 76 matchsticks and for 100 squares, 301 matchsticks

Luke - Luke drew 1 'square' and 6 'inverted C's'. As each square consists of 4 matchsticks and each 'inverted C' of 3 altogether he uses $(1\times4)+(3\times6)=4+18=22$ matchsticks
∴ When drawing n squares Luke will need 1 square and n-1 'inverted C's' ⇒ he will use $4+3(n-1)=4+3n-3=1+3n$
∴ For drawing 25 squares he will use 76 matchsticks and for 100 squares 301 matchsticks

As you can see no matter which method you choose the number of matchsticks needed is a constant. The number of matchsticks to draw n squares is always $1+3n$.

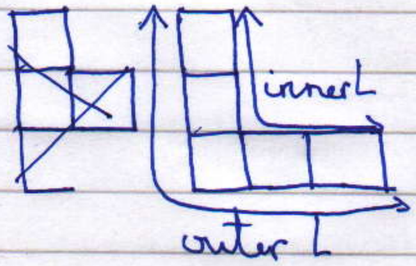
Growing rectangles

As for any rectangle the perimeter is $= 2(l+b)$, here $l=2$ and $b=n$ $\therefore p = 2(2+n)$. There are n lines per row and ~~n+1 row~~ 3 rows $\Rightarrow 3n$ lines

The number of dots on each row is $n+1$ and there are 3 rows $\Rightarrow$ there are $3(n+1)$ dots.

$\therefore$ When the width is 25, $p = 54$, ~~and~~ no of dots $= 78$ and no. of lines $= 75$ and when the width is 100, $p = 204$, and no. of dots $= 303$ and no of lines $= 300$.

L-shapes



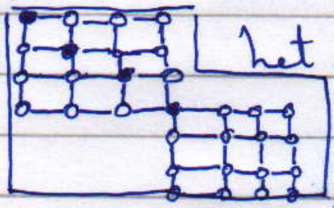
The length of the 'outer L' $= 2n$ and of the inner L $= 2(n-1)$. There are 2 remaining ~~units~~ sides left each measuring 1 unit. $\Rightarrow$ altogether the perimeter $= 2n + 2(n-1) + 2 = 2(n+n-1+1) = 2 \times 2n = 4n$.

The no. of squares is equal to $2n-1$. The reason for this is the following. Think of ~~each~~ an L as two congruent overlapping rectangles ~~each~~ with dimensions n and 1. The no of squares in each of these $= n$. So if you add their areas together you get $2n$. However, they overlap with 1 square $\Rightarrow$ that their area $= 2n-1 =$ no of squares per L shape.

The no. of lines $=$ the perimeter $+ 2(n-1) = 4n + 2n - 2 = 6n - 2 = 2(3n-1)$.

$\therefore$ For ~~Squares~~ dimensions 25 no. of lines $= 148$, no of squares $= 49$, $p = 100$. For length 100 no of lines $= 598$, no. of squares $= 199$ and $p = 400$

2 rectangles



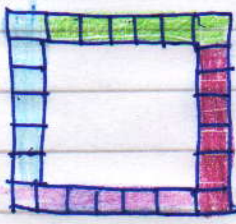
Let us treat these as two seperate squares, then the no. of dots altogether $= 2n^2$. Now, when there is an overlap of 1 dot there are $2n^2 - 1$ dots and the no. of white dots $= 2n^2 - 1 - n$.

~~When~~ In each square there is $(n-1)n$ lines

$\therefore$ Altogether there are $2n(n-1)$ lines

$\therefore$ When no of black dots $= 25$, no of white dots $= 1324$ and no of lines $= 1200$. When no. of black dots $= 100$, no of white dots $= 19999$, no of lines $= 19800$.

Square of squares

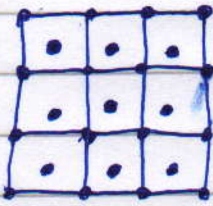


Each of these blocks contain $n-1$ squares $\Rightarrow$ there are $4(n-1)$ edge squares.
The no. of lines on the outer square $= 4n$, on the inner there are $4(n-2)$. When these are taken away there are $4(n-1)$ lines left. $\Rightarrow$ altogether there are $4n+4(n-2)+4(n-1)$

$= 4(n+n-2+n-1)$
$= 4(3n-3)$
$= 12(n-1)$

$\therefore$ When $n=25$, there are 96 edge squares and 288 lines. When $n=100$ there are 396 edge squares and 1188 lines.

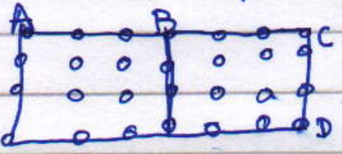
Dots and more dots



From inside the squares there are n^2 dots and on the vertices of the squares there are $(n+1)^2$ dots. $\Rightarrow$ altogether there are $n^2+(n+1)^2$ dots.
$= 2n^2+2n+1$.

Per row there are n lines and there are $n+1$ rows.
$\therefore$ From the 'row contribution' $n(n-1)$ lines. From the 'column contribution' there are again $n(n-1)$ lines $\Rightarrow$ altogether there are $2n(n-1)$ lines.
$\therefore$ When $n=25$, there are 1401 dots and 1200. When $n=100$, there are 2201 dots and 19800 lines.

Rectangle of dots.



$\overline{AB}$ uses n lines. So, the horizontal contribution $= 4n$. $\overline{CD}$ uses n lines. So, the vertical contribution $= 3n$. Altogether it uses $3n+4n = 7n$ lines.
On each row there are $2n+1$ dots and there are $n+1$ rows.
$\therefore$ There are $(2n+1)(n+1)$ dots.
$\therefore$ When $n=25$ there are 175 lines and there are 1326 dots. When $n=100$ there are 700 lines and 20301 dots.