Sticky Triangles

Method 1

Row Number

1 base form 1Δ

1 → $3 \times 1\Delta = 3$ sides (sticks)

(2 sticks) + (1 base) = 3 (total)

$$\begin{array}{r} 1 \\ +2 \\ \hline 3 \end{array}$$ bases form 3Δ

2 → $3\times3\Delta - 3\times1\Delta$ (Double Counted)

$9-3=6$ sides

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$$ bases form 5Δ

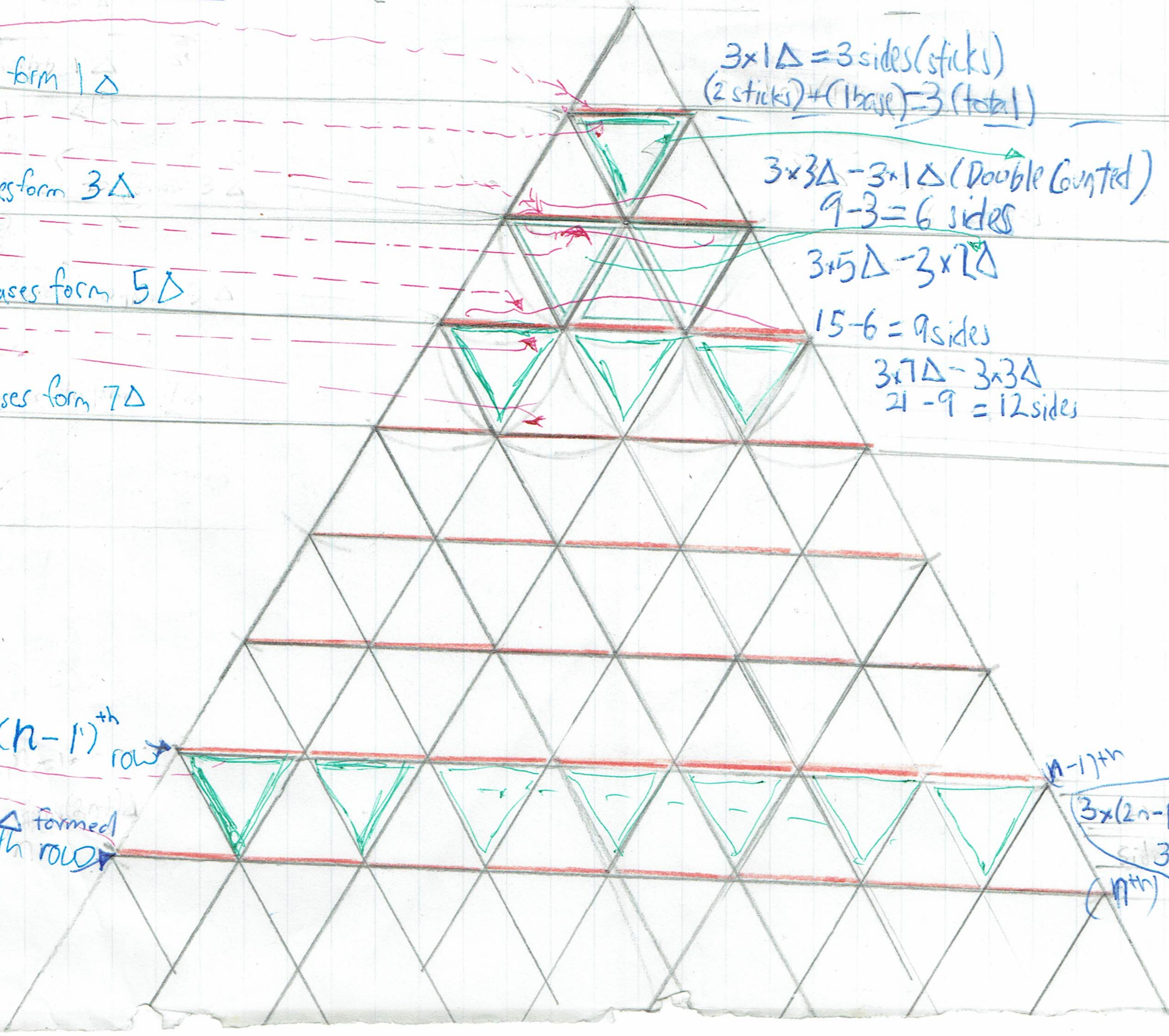
3 → $3\times5\Delta - 3\times2\Delta$

$15-6=9$ sides

$$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$$ bases form 7Δ

4 → $3\times7\Delta - 3\times3\Delta$

$21-9=12$ sides



$(n-1)^{th}$ row

$$\begin{array}{r} n-1 \\ +n \\ \hline (2n-1) \end{array}$$ Δ formed

n^{th} row

$(n-1)^{th}$

(n^{th})

$3\times(2n-1)\Delta - 3\times(n-1)\Delta$

$3(2n-1) - 3\times(n-1)$ sides

$= 3n$ sides

Table 1

Row Number	1	2	3	4	. . .	n^{th}
no. of triangles(Δ) formed in this row	1	3	5	7		$2n-1$
total Δ formed	1	$1+3$	$1+3+5$	$1+3+5+7$		$1+3+5 \dots + (2n-1)$

Table 2

Row No.	1	2	3	4	. . .	n^{th}
no. of sticks used in this row	3	6	9	12		$3n$
total no. of sticks used	3	$3+6$	$3+6+9$	$3+6+9+12$		$3+6+9 \dots + 3n$

Method 2

① Count the triangles and sticks and record the info into Table 3 and 4

Table 3

Row no.	1	2	3	4	. . . .	n^{th}
△ no.	1	1+3	1+3+5	1+3+5+7	. . . . .	1+3+5 . . . +(2n-1)

② Analyse the pattern 2×2-1 2×3-1 2×4-1 ∴ For n^{th} it is $2n-1$

Table 4

Row no.	1	2	3	4	. . . . . .	n^{th}
Stick no.	3	3+6	3+6+9	3+6+9+12	. . . . .	3+6+9+ . . (3n)

3×2 3×3 3×4

③ Analyse the pattern ∴ For n^{th} it is $3n$.

♡ Investigate the number series:

(1) Number of small triangles △

$1+3+5+7+\ldots+(2n-1)$

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad \ldots \; T_n$ represents the value of the n^{th} term

Examples

$n=6$.

$\underbrace{1+3+5+7+9+11}$

$\frac{6}{2}$ pairs of $1+11=12$

Total: $\frac{6}{2} \times 12 = 3 \times 12$

$= 36$

if n is even

$\underbrace{1+3+5+7+\ldots+(2n-1)}_{1+(2n-1)}$

$=$

$\frac{n}{2}$ pairs of $2n$

total $\frac{n}{2} \times 2n = n^2$

$n=7$

$\underbrace{1+3+5+7+9+11}+13$

$\frac{6}{2}$ pairs of $1+11=12$

Total: $36+13 = 49$

if n is odd

$1+3+5+7+\ldots+T_{n-1}+T_n$

$2(n-1)-1 \quad (2n-1)$

$1+T_{n-1}$

$1+2(n-1)-1$

$=1+2n-2-1$

$=2n-2$

$\frac{n-1}{2}$ pairs of $2\times(n-1)$

total: $\frac{n-1}{2} \times 2 \times (n-1) + (2n-1)$

total $=(n-1)^2+(2n-1)$

② Number of sticks ♡

$3+6+9+\ldots+3n$

$T_1 \quad T_2 \quad T_3$

T_n represents the value of the n^{th} term

Examples

$n=4$

$\underbrace{3+6+9+12}_{3+12=15}$

$\frac{4}{2}$ pairs of 15

Total: $\frac{4}{2}\times15$
$=2\times15$
$=30$

if n is even

$\underbrace{3+6+9+12+\ldots+3n}_{3+3n}$

$\frac{n}{2}$ pairs of $=3\times(n+1)$

Total $\frac{n}{2}\times3\times(n+1)$

$n=5$

$3+6+9+12+15$

$\frac{4}{2}$ pairs of $3+12=15$

Total: $\frac{4}{2}\times15+15$
$=30+15$
$=45$

If n is odd

$3+6+9+\ldots+T_{n-1}+T_n$

$3(n-1) \quad 3n$

$\frac{n-1}{2}$ pairs of $3+3(n-1)$

Total: $\frac{n-1}{2}\times(3+3(n-1))+3n$

Ong Ci Hui Minh Ngoc
Grade 5 Kelvin Grove State College
Brisbane, Australia

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