$$\frac{1}{\sqrt{1}+\sqrt{2}} \times \frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}} = \frac{\sqrt{1}-\sqrt{2}}{1-2} = \frac{\sqrt{1}-\sqrt{2}}{-1}$$

$\sqrt{2}$ cancel one another out

$$\frac{\sqrt{2}-\sqrt{3}}{-1}$$

...

Each root cancels itself out except the $\sqrt{1}$ & $-\sqrt{100}$.

$$\frac{\sqrt{1}-\sqrt{100}}{-1} = \frac{1-10}{-1} = \frac{-9}{-1} = 9$$

To solve this problem you must first rationalise the denominator. This process will always give you the first root subtract the second root over negative one. Each root will cancel itself out except $\sqrt{1}$ (root 1) and $-\sqrt{100}$ (negative root 100). The problem can then be solved:

Square root of $1 = 1$ Square root of $100 = 10$

$1 - 10 = -9$ $\quad -9 \div -1 = 9$